



## Special Right Triangles and Trigonometry

### Lesson Synopsis:

In this lesson, special right triangles are explored using concrete modeling and students' previous knowledge of the Pythagorean Theorem. Students apply their knowledge of special right triangles in order to solve meaningful problems. In addition, students develop an understanding of right triangle trigonometry by developing trigonometric ratios. Trigonometric ratios are developed using students' previous knowledge of right triangles, dilations, and similarity. Students apply their knowledge of trigonometric ratios in order to solve meaningful problems and verify special right triangle theorems.

### TEKS:

**G.5** *Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems.*

**G.5A** Use numeric and geometric patterns to develop algebraic expressions representing geometric properties.

**G.5B** Use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures ~~and solids~~, and angle relationships in polygons ~~and circles~~.

**G.5D** Identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

**G.11** *Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems.*

**G.11C** Develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods.

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## GETTING READY FOR INSTRUCTION

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### Performance Indicator(s):

- Develop the special right triangle relationships ( $45^\circ$ - $45^\circ$ - $90^\circ$  and  $30^\circ$ - $60^\circ$ - $90^\circ$ ) and trigonometric ratios, and apply them to determine the solution to a problem situation involving right triangles. (G.5A, G.5B, G.5D; G11.C)

**ELPS:** 1E, 1H, 2E, 2I, 3H, 4F, 5G

### KEY Understandings and Guiding Questions:

- $45^\circ$ - $45^\circ$ - $90^\circ$  and  $30^\circ$ - $60^\circ$ - $90^\circ$  right triangles have special relationships that can be used to find missing measures.
  - How are the side lengths of a  $45^\circ$ - $45^\circ$ - $90^\circ$  Triangle related?
  - How are the side lengths of a  $30^\circ$ - $60^\circ$ - $90^\circ$  Triangle related?
  - How can these relationships be used to solve special right triangles?
  - What role does similarity play in special right triangles?
- Problem situations involving right triangles can be solved using trigonometric ratios (sine, cosine, tangent).
  - What are trigonometric ratios?
  - How are the sine, cosine, and tangent ratios formed?
  - How can trigonometric ratios be used to solve right triangles?
  - What role does similarity play in trigonometric ratios?


### Vocabulary of Instruction:

- |   |                       |                       |
|---|-----------------------|-----------------------|
| • special right triangles                       | • adjacent            | • cosine              |
| • $45^\circ$ - $45^\circ$ - $90^\circ$ triangle | • hypotenuse          | • tangent             |
| • $30^\circ$ - $60^\circ$ - $90^\circ$ triangle | • trigonometric ratio | • angle of elevation  |
| • opposite                                      | • sine                | • angle of depression |

### Materials:

- graphing calculator
- ruler

## Resources:

-  **STATE RESOURCES:**
  - **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments  
<http://www.utdanacenter.org/mathtoolkit/index.php>
  - **MTC Geometry:** Student Lessons – Investigation 1 Similarity, Explore/Explain, Act. 8 (45-45-90 Triangles Part 1), Act. 9 (45-45-90 Triangles Part 2), Act. 10 (Know Your Hypotenuse), Act. 12 (Long Leg, Short Leg), Act. 13 (Pythagoras and Sine), Act. 14 (Cosine and Tangent)  
<http://www.tea.state.tx.us/math/index.html>
  - **TEXTEAMS: High School Geometry: Supporting TEKS and TAKS:** 5.0 Investigating Right Triangle Ratios, 5.1, Act. 1 (Right Triangle Trig and the Unit Circle); 6.0 Student Activity – The Tangent Ratio, 6.1, Act. 1 (The Tangent Ratio); 7.0 Foundations and Trusses, 7.1, Act. 1 (Laying Out a Foundation), 7.2, Act. 2 (Building Trusses)

## Advance Preparation:

1. Handout: **It's All About the Roof Pitch** (1 per student)
2. Transparency: **It's All About the Roof Pitch** (1 per teacher)
3. Handout: **Exploring Special Right Triangles** (1 per student)
4. Handout: **Special Right Triangles** (1 per student)
5. Handout: **Applications of Special Triangles** (1 per student)
6. Handout: **Exploring Trigonometric Ratios** (1 per student)
7. Handout: **Trigonometric Ratios** (1 per student)
8. Handout: **Applications of Trigonometry** (1 per student)
9. Handout: **Roof Pitch Revisited** (1 per student)

## Background Information:

This lesson uses previously taught properties of the right triangle, such as the Pythagorean Theorem, to develop further properties including those of the special right triangles. In addition, trigonometric ratios are developed using dilations and similarity. As a result, students should be familiar with these concepts and proportional reasoning in order to connect to its extension to trigonometry.

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## GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

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Instructors are encouraged to supplement, and substitute resources, materials, and activities to differentiate instruction to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this planning document is located at [www.cscope.us/sup\\_plan\\_temp.doc](http://www.cscope.us/sup_plan_temp.doc). If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource in your district Curriculum Developer site for future reference.

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## INSTRUCTIONAL PROCEDURES

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### Instructional Procedures

#### ENGAGE

1. Distribute the handout: **It's All About the Roof Pitch** to each student.
2. Have students work individually to complete **It's All About the Roof Pitch**.
3. Debrief the activity using the transparency: **It's All About the Roof Pitch**. Frame the discussion around question 3 which is listed below as a facilitation question.  
Facilitation Questions:
  - **Do you think there is a relationship between the ratio of the sides of a right triangle and the measures of the acute angles? Why or why not?** *At this point, students will most likely be divided in opinion over this question, and that is expected. This question is meant to set*

### Notes for Teacher

**NOTE:** 1 Day = 50 minutes  
**Suggested Day 1 (1/4 day)**

#### MATERIALS

- Handout: **It's All About the Roof Pitch** (1 per student)
- Transparency: **It's All About the Roof Pitch** (1 per teacher)

#### TEACHER NOTE

The purpose of this activity is to introduce students to the idea of an isosceles right triangle (the 45°-45°-90°

## Instructional Procedures

*the stage for the rest of the lesson cycle as the ratios of sides in right triangles play a KEY role in developing the 45°-45°-90° Theorem and 30°-60°-90° Theorem, and trigonometric reasoning.*

- **How were you able to determine the measures of the acute angles in the right triangle you drew?** *Since the triangle is Isosceles, the base angles (the acute angles in the right triangle) are congruent. (Isosceles Triangle Theorem)*
- **How were you able to find the length of the hypotenuse?** *The Pythagorean Theorem.*
- **Describe your procedure for finding the length of the rafter given the span of the room.** *Since the roof has 12:12 pitch (the rise and run are the same), the length of the legs of the isosceles right triangle is half the room span or 10ft. By Pythagorean Theorem, the length of the rafter is  $10\sqrt{2}$  ft. or about 14.14 ft.*

## EXPLORE 1

1. Distribute the handout: **Exploring Special Right Triangles** to each student.
2. Explain the rectangular grid on p. 1 to students, focusing on the distance between points. Work the first triangle with students in whole group. Have students work with a partner to complete the remainder of the 45°-45°-90° section on pp. 1-2.
3. Debrief the 45°-45°-90° section in whole-group discussion.
4. Explain the isometric grid on p. 3 to students, focusing on the distance between points. Work the first triangle with students in whole group. Have students work with a partner to complete the remainder of the 30°-60°-90° section.
5. Debrief the 30°-60°-90° section in whole-group discussion.

If time is a factor, the 30°-60°-90° section can be completed as homework and debriefed at the beginning of the next class period.

## Notes for Teacher

Triangle) by posing a problem that can be solved with the Pythagorean Theorem. The Pythagorean Theorem plays a KEY role in developing the special right triangle theorems (45°-45°-90° Theorem and 30°-60°-90° Theorem).



### STATE RESOURCES

**Mathematics TEKS Toolkit:** Clarifying Activity/Lesson/Assessment may be used to reinforce these concepts or used as alternate activities.

Suggested Day 1 (3/4 day)

### MATERIALS

- Handout: **Exploring Special Right Triangles** (1 per student)
- ruler
- graphing calculator

### TEACHER NOTE

The purpose of this activity is to give students the opportunity to use the Pythagorean Theorem to build knowledge of 45°-45°-90° Triangles and 30°-60°-90° Triangles. Students construct triangles using grid paper and record data in tables in order to discover the ratio of side lengths for the special right triangles.

Pay particular attention to the directions given to students in questions 3 and 11. The intent of question 3 is that students use the length of the hypotenuse determined from the 1<sup>st</sup> triangle to predict the subsequent hypotenuse by comparing this length to the diagonal spacing between the points on the grid paper. Some students may revert to the Pythagorean Theorem in order to find the length of the hypotenuse. Redirect students as necessary. The intent of question 11 is similar in that students should use the value of the long leg determined from the 1<sup>st</sup> triangle and the vertical spacing between the points on the grid to predict the next values. Redirect students as necessary.



### STATE RESOURCES

**MTC Geometry:** Student Lessons – Investigation 1 Similarity,

## Instructional Procedures

### EXPLAIN 1

- Debrief the handout: **Exploring Special Right Triangles** with students. Use the data generated by the activity to facilitate a discussion of special right triangles. Some facilitation questions are listed below.  
Facilitation Questions:  
**45°-45°-90° Triangles**
  - What is the length of the hypotenuse of the isosceles right triangle with leg length 1?**  $\sqrt{2}$
  - How do you know this initial length of the hypotenuse?** *Pythagorean Theorem.*
  - What does the length of the hypotenuse correspond to on the grid paper?** *The diagonal distance between two points.*
  - Since the distance between two diagonal points is  $\sqrt{2}$ , how can this information be used to predict the hypotenuse length of an isosceles right triangle (45°-45°-90° Triangle) with leg length 2?**  
*Since the diagonal distance between two points is  $\sqrt{2}$ , each subsequent diagonal point would add an additional  $\sqrt{2}$  of length to the hypotenuse. Therefore, the length of the hypotenuse of an isosceles right triangle with leg length 2 is  $\sqrt{2} + \sqrt{2}$  or  $2\sqrt{2}$ .*
  - Based on the data in your table, were you able to write a ratio for the side lengths of an isosceles right triangle (45°-45°-90° Triangle)? If so, what is the ratio?** *Yes, the leg length to leg length to hypotenuse ratio is  $x : x : x\sqrt{2}$ .*
  - How can this ratio be used to make predictions about other 45°-45°-90° Triangles?** *Given one of the sides of the triangle, proportional reasoning can be used to find the other sides.*
- 30°-60°-90° Triangles**
  - Recall that on the grid paper, the horizontal and diagonal spaces between two points represent one unit of length. How did you use this information to find the length of the long leg of the right triangle?** *The length of the short leg and the length of the hypotenuse were known, so by Pythagorean Theorem, the length of the long leg is  $\sqrt{3}$ .*
  - What does the length of the long leg correspond to on the grid paper?** *The vertical distance between two points.*
  - What are the measures of the acute angles of the triangles you drew on the grid paper? How do you know?** *30° and 60°. Since the points on the grid paper form equilateral triangles, the measure of one of the acute angles of the triangle drawn is 60°, the other must be 30° because the acute angles of a right triangle are complementary (or Triangle Angle Sum Theorem).*
  - Since the distance between two vertical points is  $\sqrt{3}$ , how can this information be used to predict the length of the longer leg of the 30°-60°-90° Triangle with short leg length 2?** *Each subsequent vertical point adds a length of  $\sqrt{3}$  to the length of the long leg;*

## Notes for Teacher

Explore/Explain, Act. 8 (45-45-90 Triangles Part 1), Act. 9 (45-45-90 Triangles Part 2), Act. 10 (Know Your Hypotenuse), Act. 12 (Long Leg, Short Leg) may be used to reinforce these concepts or used as alternate activities.

### Suggested Day 2 (1/2 day)

#### MATERIALS

- Handout: **Special Right Triangles** (1 per student)

#### TEACHER NOTE

The purpose of this activity is to summarize the properties of special right triangles.

#### TEACHER NOTE

Not all students will understand operations with radicals. The behavior of adding  $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$  is like adding  $x + x = 2x$ .

## Instructional Procedures

therefore, if the short leg is 2 units long, the long leg is  $2\sqrt{3}$  units long.

- According to the data in your table, how does the length of the hypotenuse in the  $30^\circ$ - $60^\circ$ - $90^\circ$  Triangle compare to the length of the short leg? *The hypotenuse is twice as long as the short leg.*
- Based on the data in your table, were you able to write a ratio for the side lengths of a  $30^\circ$ - $60^\circ$ - $90^\circ$  Triangle? If so, what is the ratio?

*The short leg to hypotenuse to long leg ratio is  $x : 2x : x\sqrt{3}$ .*

- How can this ratio be used to make predictions about other  $30^\circ$ - $60^\circ$ - $90^\circ$  Triangles? *Given one of the sides of the triangle, proportional reasoning can be used to find the other sides.*

- Distribute the handout: **Special Right Triangles** to students. Go over notes and examples in whole group. Some of the problems can be worked as Guided Practice in pairs and debriefed in whole group.

## Notes for Teacher

### ELABORATE 1

- Distribute the handout: **Applications of Special Triangles** to each student.
- Have students continue to work in pairs or small groups on the handout. This can be completed as homework, if necessary.

### Suggested Day 2 (1/2 day)

#### MATERIALS

- Handout: **Applications of Special Triangles** (1 per student)

#### TEACHER NOTE

The purpose this activity is to give students the opportunity to apply their knowledge of special right triangles in a problem solving setting. For example, students examine problems about area and perimeter, or real-world problems that can be solved using special right triangles.

### EXPLORE 2

- Debrief handout: **Applications of Special Triangles** by sharing answers in whole-group instruction.
- Distribute the handout: **Exploring Trigonometric Ratios** to each student.
- Have students work with a partner to complete Part I of **Exploring Trigonometric Ratios**.
- When students have completed Part I, have each pair group with another pair to share answers and discuss results.
- Have students work in the group of four to complete Part II. Make sure each student has a calculator.
- Students may complete Part II as homework, if they have access to graphing calculators.

Part II can be completed at the beginning of the next class period if necessary.

### Suggested Day 3

#### MATERIALS

- Handout: **Exploring Trigonometric Ratios** (1 per student)
- graphing calculators

#### TEACHER NOTE

This activity is divided into two parts. The purpose of Part I is to give students the opportunity to develop the trigonometric ratios using their previous knowledge of dilations and similar triangles. Students create and compare the Sine, Cosine, and Tangent ratios by dilating a 3, 4, 5 right triangle, and special right triangles. The purpose of Part II is to give students the opportunity to see the role technology plays in using trigonometric ratios. Students use the graphing calculator in order to develop a deeper understanding of trigonometric ratios and their uses.



#### STATE RESOURCES

MTC Geometry: Student Lessons –

## Instructional Procedures

## Notes for Teacher

Investigation 1 Similarity, Explore/Explain, Act. 13 (Pythagoras and Sine), Act. 14 (Cosine and Tangent) may be used to reinforce these concepts or used as alternate activities.

### TEXTEAMS: High School Geometry: Supporting TEKS and TAKS: 5.0

Investigating Right Triangle Ratios, 5.1, Act. 1 (Right Triangle Trig and the Unit Circle); 6.0 Student Activity – The Tangent Ratio, 6.1, Act. 1 (The Tangent Ratio); 7.0 Foundations and Trusses, 7.1, Act. 1 (Laying Out a Foundation), 7.2, Act. 2 (Building Trusses) may be used to reinforce these concepts or used as alternate activities.

## EXPLAIN 2

1. Debrief the handout: **Exploring Trigonometric Ratios** with students. Use the data in the activity to guide a discussion of trigonometric ratios. Some facilitation questions are listed below.

### Facilitation Questions:

- **What do the terms *opposite* and *adjacent* mean in relation to the sides of a right triangle?** *Given one of the two acute angles of a right triangle, the opposite side is across from the given angle; the adjacent side is next to the given angle.*
- **How is the sine ratio formed?**  $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$
- **How is the cosine ratio formed?**  $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$
- **How is the tangent ratio formed?**  $\tan = \frac{\text{opposite}}{\text{adjacent}}$
- **How did the sine (cosine and tangent) ratios for similar right triangles compare for angles of the same value?** *The sine ratios were equal. (The cosine ratios were equal. The tangent ratios were equal). Why do you think this is true? Since the triangles are similar, the corresponding side lengths are proportional; therefore, corresponding trigonometric ratios are equal.*
- **How does the size or area of the triangle influence the value of the trigonometric ratios?** *It does not influence the trig ratios at all; only the value of the acute angle for which a trig ratio is formed will influence the value of the trig ratio.*
- **Suppose you know the value of an acute angle of a right triangle but none of the side lengths. How can you find values for the Sine, Cosine, and Tangent of the angle? Explain.** *The values of the trig ratios are stored in the graphing calculator.*
- **How do the trig ratios for 30°, 45°, and 60° that you found using the graphing calculator compare to those you found using the side lengths of the special right triangles? Explain.** *They are the same. The calculator represents the ratios in a decimal form that is equivalent to the ratio.*
- **Given two side lengths of a right triangle, how can you determine information about the angles of the triangle? Explain.** *Given two side lengths, one of the trig ratios for a particular acute angle can be formed. Using the inverse trig functions of the graphing calculator returns the angle value.*

## Suggested Day 4

### MATERIALS

- Handout: **Trigonometric Ratios** (1 per student)

### TEACHER NOTE

The purpose of this activity is to summarize the properties of the trigonometric ratios.

### TEACHER NOTE

Student understanding of labeling sides of a triangle in order to determine trig ratios is critical. Make sure students understand that although the *hypotenuse* is always identified the same way (longest side, across from the right angle); terms like *opposite* and *adjacent* are relative to a particular acute angle in the right triangle.

### TEACHER NOTE

It is important for students to realize that the value of a particular trig ratio is based on the side lengths of a *right* triangle. Students sometimes attempt to apply trig ratios inappropriately in triangles that do not contain a right angle.

Instructional Procedures	Notes for Teacher
<ol style="list-style-type: none"> <li>Distribute the handout: <b>Trigonometric Ratios</b> to each student.</li> <li>Demonstrate example problems from <b>Trigonometric Ratios</b>.</li> <li>Have students complete Practice Problems from <b>Trigonometric Ratios</b>. These may be assigned for homework, if necessary.</li> </ol>	
ELABORATE 2	Suggested Day 5
<ol style="list-style-type: none"> <li>Distribute handout <b>Applications of Trigonometry</b>. Model example problems for Applications of Trigonometry. Give special attention to question #3 involving angles of elevation and depression.</li> <li>Have students complete the Practice Problems from <b>Applications of Trigonometry</b>. These may be assigned for homework, if necessary.</li> </ol>	<p><b>MATERIALS</b></p> <ul style="list-style-type: none"> <li>Handout: <b>Applications of Trigonometry</b> (1 per student)</li> <li>graphing calculator</li> </ul> <p><b>TEACHER NOTE</b> The purpose of this activity is to give students an opportunity to apply their knowledge of trigonometric ratios in a problem-solving setting.</p> <p><b>TEACHER NOTE</b> Example problems are given in <b>Applications of Trigonometry</b> for the teacher to model. The example problems not only model using the trigonometric ratios in problem solving, but also introduce the notion of an <i>angle of elevation</i> and an <i>angle of depression</i>. This is new vocabulary for students and it is important that they grasp the meaning of these phrases before tackling the problems.</p> <p><b>TEACHER NOTE</b> It is important for students to realize that the <i>angle of elevation</i> or the <i>angle of depression</i> may not always be part of the right triangle that is being solved. Most likely, students will be required to draw upon their knowledge of special angle pairs formed when parallel lines are cut by a transversal in order to use <i>angles of elevation</i> or <i>angles of depression</i> to discern information about right triangles.</p>
EVALUATE	Suggested Day 6
<ol style="list-style-type: none"> <li>Debrief handout: <b>Applications of Trigonometry</b> in whole-group discussion as a review before the assessment.</li> <li>Distribute the handout: <b>Roof Pitch Revisited</b> to each student.</li> <li>Have students complete <b>Roof Pitch Revisited</b> individually as an assessment of student conceptual understanding.</li> </ol>	<p><b>MATERIALS</b></p> <ul style="list-style-type: none"> <li>Handout: <b>Roof Pitch Revisited</b> (one copy per student)</li> </ul> <p><b>TEACHER NOTE</b> This activity should be completed independently to assess student knowledge of the concepts taught in the lesson.</p> <p><b>TEACHER NOTE</b> In this activity, students revisit a situation that is similar to the one posed</p>

## Instructional Procedures

## Notes for Teacher

in the opening activity. Given a problem situation, students identify special right triangle relationships in order to solve problems, and justify their solutions using their knowledge of trigonometric ratios.



## TAKS CONNECTION

Grade 11 TAKS 2003 #20,30

Grade 11 TAKS 2004 #12,19,46

Grade 11 July TAKS 2004 #7,17,21,38

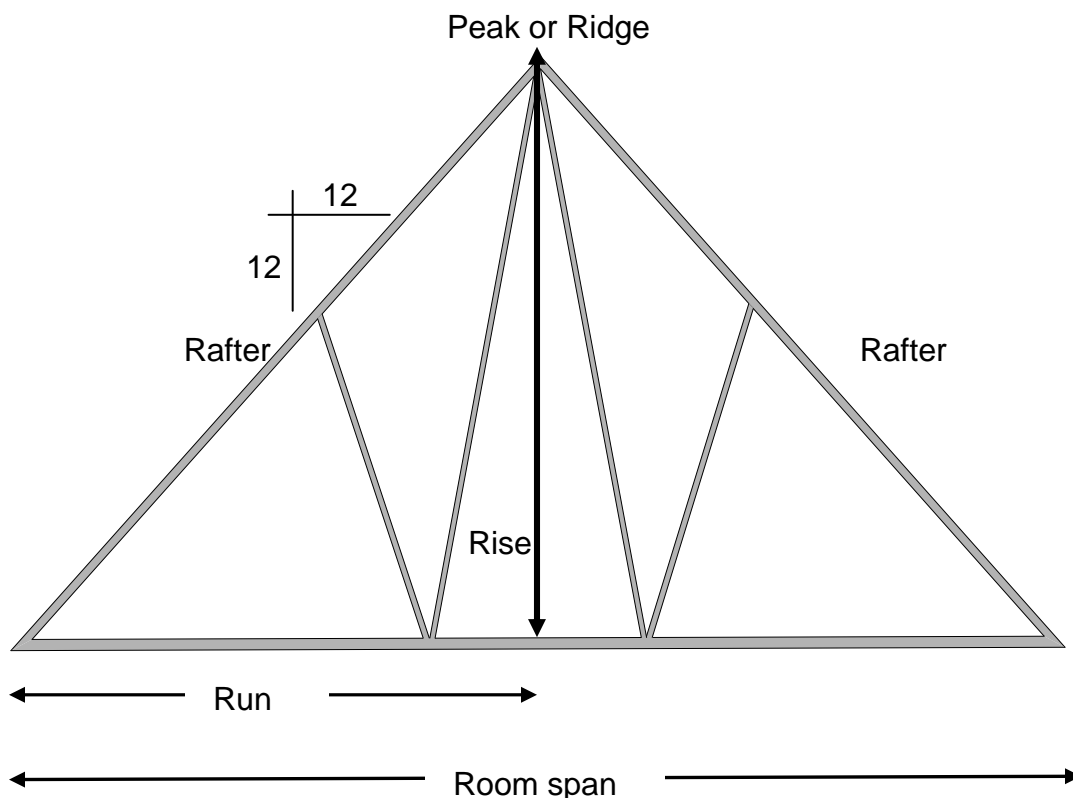
Grade 11 TAKS 2006 #15,39

Grade 11 July TAKS 2006 #10,46



## It's All About the Roof Pitch **KEY**

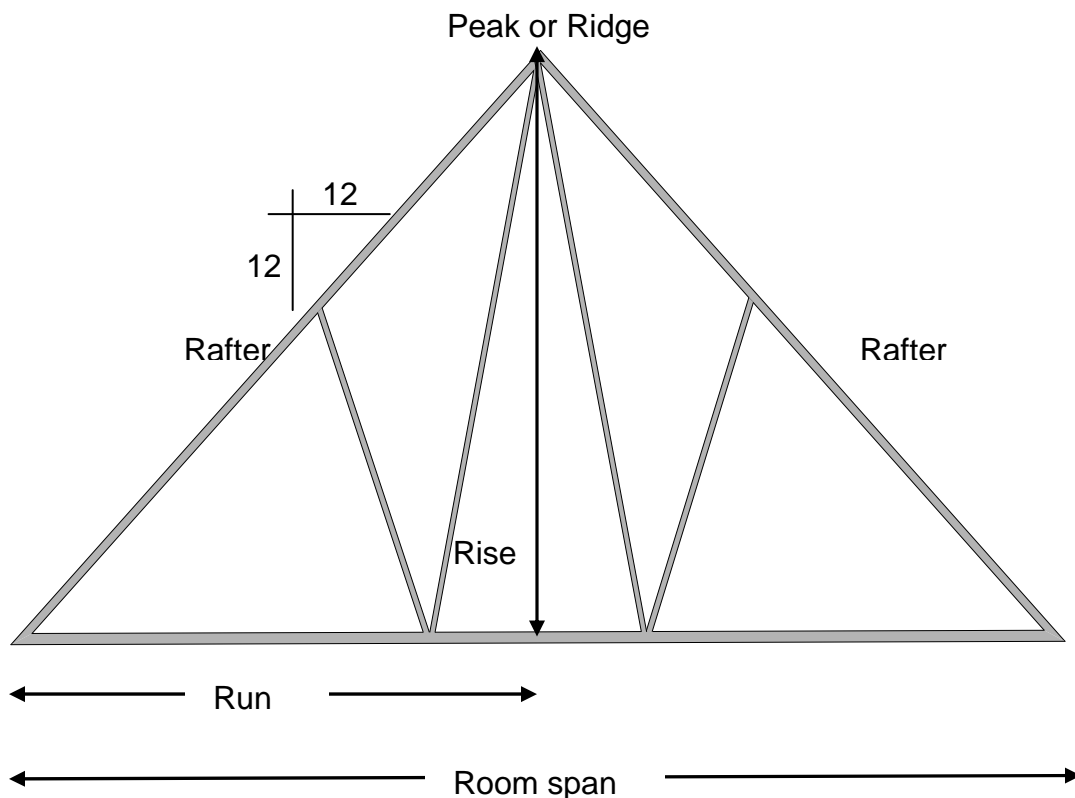
Below is typical of what you would see on a drawing used for construction of the framing members of the roof section of a house or building. The pitch or “slope” of the roof is expressed as a ratio of “rise” to “run”. The notation in the drawing identifies the roof pitch as 12/12 meaning the roof will rise 12 inches for each 12 inches of span across the ceiling joist until reaching the peak or ridge of the roof.



- To simplify the drawing above, sketch a right triangle so that the vertical leg (rise) is 12 units and the horizontal leg (run) is 12 units. How can you find the length of the hypotenuse?  
**By Pythagorean Theorem the hypotenuse is  $12\sqrt{2}$ .**
- Since both legs of the triangle you sketched are 12 units long, what kind of triangle is the right triangle? Can you use this information to find the measures of the acute angles in the triangle?  
**It is an isosceles right triangle. The acute angles both have a measure of  $45^\circ$  by the Isosceles Triangle Theorem.**
- Do you think there is a relationship between the ratio of the sides of a right triangle and the measures of the acute angles? Why or why not?  
**Answers will vary. See discussion in instructional procedures.**
- Suppose the span of a room is 20 ft., how can the information in the drawing and from question 1 be used to find the height of the ridge and the length of the rafter?  
**Using the Pythagorean Theorem, the rise and run of the triangle would both be 10' or half the span of the room; therefore, the rafter length is  $10\sqrt{2}$ .**

## It's All About the Roof Pitch

Below is typical of what you would see on a drawing used for construction of the framing members of the roof section of a house or building. The pitch or “slope” of the roof is expressed as a ratio of “rise” to “run”. The notation in the drawing identifies the roof pitch as 12/12 meaning the roof will rise 12 inches for each 12 inches of span across the ceiling joist until reaching the peak or ridge of the roof.

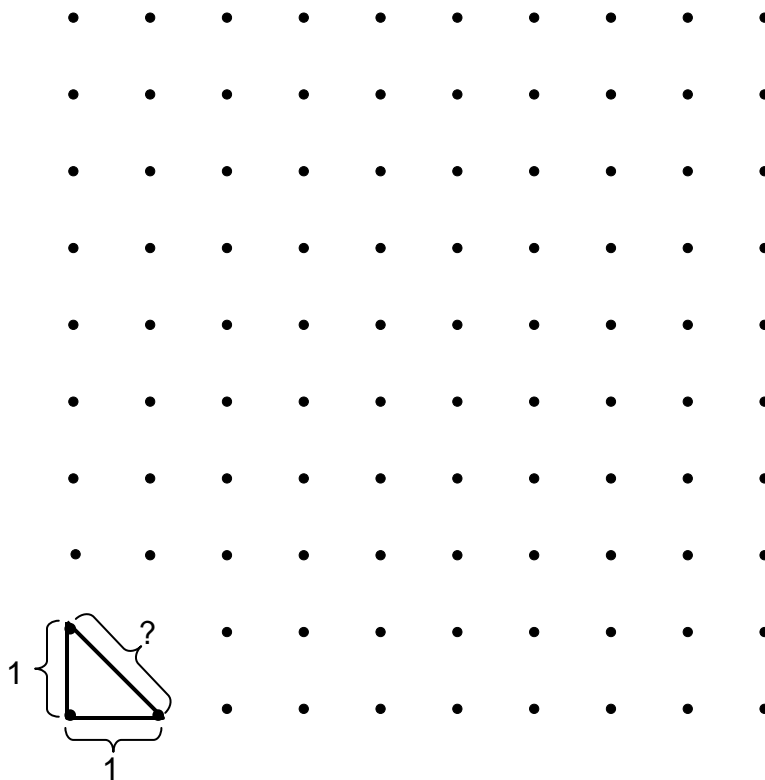


1. To simplify the drawing above, sketch a right triangle so that the vertical leg (rise) is 12 units and the horizontal leg (run) is 12 units. How can you find the length of the hypotenuse?
2. Since both legs of the triangle you sketched are 12 units long, what kind of triangle is the right triangle? Can you use this information to find the measures of the acute angles in the triangle?
3. Do you think there is a relationship between the ratio of the sides of a right triangle and the measures of the acute angles? Why or why not?
4. Suppose the span of a room is 20 ft., how can the information in the drawing and from question 1 be used to find the height of the ridge and the length of the rafter?

## Exploring Special Right Triangles (pp. 1 of 5) **KEY**

### 45°-45°-90° Triangles

Below is an orthographic or rectangular grid. All points horizontally and vertically are equal distance apart.



- Let the lower left corner of the grid be the vertex of a right angle. Draw a right triangle so that each leg is one unit long. Since each leg is one unit long, what type of right triangle did you draw? What are the measures of the acute angles of the right triangle? How do you know? **The first triangle is drawn. The triangle drawn is an isosceles right triangle. The acute angles both have a measure of 45° by the Isosceles Triangle Theorem.**
- Notice the length of the hypotenuse is the diagonal distance between two points. How long is the hypotenuse (express your answer as a radical)? How did you find your answer?  
**The hypotenuse is  $\sqrt{2}$  units long by the Pythagorean Theorem.**
- Draw another right triangle with legs of two units using the same right angle as before. Use the value of the hypotenuse in the first triangle to determine the length of the hypotenuse in this triangle.  
 **$\sqrt{2} + \sqrt{2}$  or  $2\sqrt{2}$  units long.**

**Note: Copier distortions may not make the grid exact and could give measurement errors. Tell students they will assume that the grid is exact with isosceles right triangles.**

## Exploring Special Right Triangles (pp. 2 of 5) **KEY**

4. Draw another right triangle with legs of three units using the same right angle as before. Use the value of the hypotenuse in the first triangle to determine the length of the hypotenuse in this triangle.

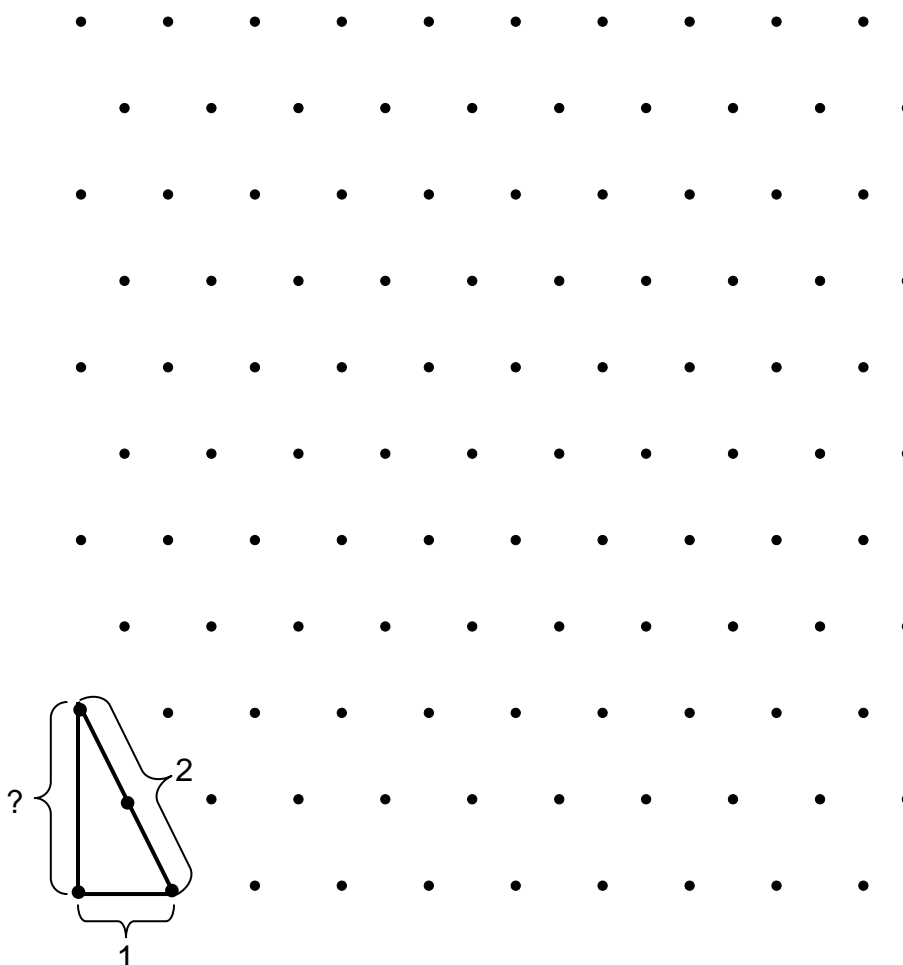
45°-45°-90° Triangle			
Length of 1st leg	Length of 2 <sup>nd</sup> leg	Length of Hypotenuse	Ratio of 1 <sup>st</sup> leg to 2 <sup>nd</sup> leg to hypotenuse
1	1	$\sqrt{2}$	1 : 1: $\sqrt{2}$
2	2	$2\sqrt{2}$	2: 2: $2\sqrt{2}$
3	3	$3\sqrt{2}$	3: 3: $3\sqrt{2}$
4	4	$4\sqrt{2}$	4: 4: $4\sqrt{2}$
5	5	$5\sqrt{2}$	5: 5: $5\sqrt{2}$
6	6	$6\sqrt{2}$	6: 6: $6\sqrt{2}$

5. Based on your data in the table above, write a conjecture about the length of the hypotenuse given the length of a leg of a 45°-45°-90° triangle.  
In a 45°-45°-90° triangle, the length of the hypotenuse is the length of the leg times  $\sqrt{2}$ .
6. Test your conjecture. If an isosceles right triangle has legs equal to eight units, predict the hypotenuse. Check your prediction by drawing the figure on the grid.  
See students' figures.
7. Suppose each leg of a 45°-45°-90° triangle has a length of  $x$ . Express the ratio of the lengths of the sides of the triangle in the form 1<sup>st</sup> leg: 2<sup>nd</sup> leg: hypotenuse.  
 $x$ :  $x$ :  $x\sqrt{2}$
8. Use your conjecture from question 5 and your ratio above to find the side lengths of a 45°-45°-90° Triangle given a leg length of 25.  
25: 25:  $25\sqrt{2}$

## Exploring Special Right Triangles (pp. 3 of 5) **KEY**

### 30°-60°-90° Triangles

Below is an isometric grid. The points form equilateral triangles; therefore the horizontal and diagonal distances between points are equal.



9. Let the lower left corner of the grid be the vertex of a right angle. Draw a right triangle so that the vertical leg extends to the first point above the vertex of the right angle and so that the horizontal leg extends to the first point to the right of the vertex of the right angle. Since the dots of the isometric grid form equilateral triangles, what is the measure of one of the acute angles of the right triangle in your drawing? What is the measure of the other acute angle? Explain your reasoning. **The first triangle is drawn. One of the acute angles has a measure of 60° since the points on the grid paper form equilateral/equiangular triangles. The other acute angle has a measure of 30° by the Triangle Angle Sum Theorem.**

**Note: Copier distortions may not make the grid exactly isometric and could give measurement errors. Tell students they will assume that the grid is isometric with equilateral triangles.**

## Exploring Special Right Triangles (pp. 4 of 5) **KEY**

10. Since the horizontal and diagonal distances between the points are equal, how long is the short leg of your right triangle? How long is the hypotenuse? Use this information to find the longer leg of the right triangle (express your answer as a radical). How did you find your answer?

The short leg has a length of 1 unit. The hypotenuse has a length of 2 units. The longer leg has length  $\sqrt{3}$  by the Pythagorean Theorem.

11. Extend each leg of your right triangle to the next point on the grid paper and draw the hypotenuse of another right triangle. How long is the hypotenuse? Use the value of the length of the long leg from the first triangle to determine the length of the long leg in the second triangle.

Hypotenuse: 4 units long, Long leg:  $2\sqrt{3}$ .

12. Repeat the above procedure by extending the legs of the triangle to the next point on the grid paper. Find the lengths of the sides of the triangle. Record your answers in the table below.

30°-60°-90° Triangle			
Length of short leg	Length of Hypotenuse	Length of long leg	Ratio of short leg to hypotenuse to long leg
1	2	$\sqrt{3}$	1: 2: $\sqrt{3}$
2	4	$2\sqrt{3}$	2: 4: $2\sqrt{3}$
3	6	$3\sqrt{3}$	3: 6: $3\sqrt{3}$
4	8	$4\sqrt{3}$	4: 8: $4\sqrt{3}$
5	10	$5\sqrt{3}$	5: 10: $5\sqrt{3}$
6	12	$6\sqrt{3}$	6: 12: $6\sqrt{3}$

13. Based on your data in the table above, write a conjecture about the length of the hypotenuse and the length of the long leg given the length of the short leg of a 30°-60°-90° Triangle.

The length of the hypotenuse is twice the length of the short leg. The length of the long leg is the length of the short times  $\sqrt{3}$ .

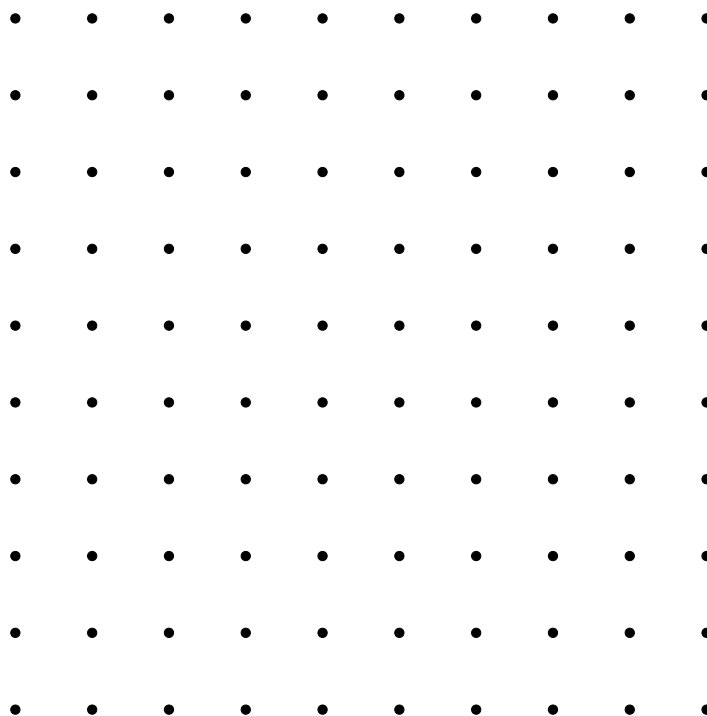
## Exploring Special Right Triangles (pp. 5 of 5) **KEY**

14. Test your conjecture. If the short leg of a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle has length five, find the length of the hypotenuse and the length of the long leg. Check your prediction by drawing the figure on the grid.  
Hypotenuse: 10 units.  
Long leg:  $5\sqrt{3}$  units.
15. Suppose the short leg of a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle has a length of  $x$ . Express the ratio of the lengths of the sides of the triangle in the form *short leg: hypotenuse: long leg*.  
 $x: 2x: x\sqrt{3}$
16. Use your conjecture from question 13 and your ratio above to find the side lengths of a  $30^\circ$ - $60^\circ$ - $90^\circ$  Triangle given a leg length of 25.  
 $25: 50: 25\sqrt{3}$

## Exploring Special Right Triangles (pp. 1 of 5)

### 45°-45°-90° Triangles

Below is an orthographic or rectangular grid. All points horizontally and vertically are equal distance apart.



1. Let the lower left corner of the grid be the vertex of a right angle. Draw a right triangle so that each leg is one unit long. Since each leg is one unit long, what type of right triangle did you draw? What are the measures of the acute angles of the right triangle? How do you know?
2. Notice the length of the hypotenuse is the diagonal distance between two points. How long is the hypotenuse (express your answer as a radical)? How did you find your answer?
3. Draw another right triangle with legs of two units using the same right angle as before. Use the value of the hypotenuse in the first triangle to determine the length of the hypotenuse in this triangle.



## Exploring Special Right Triangles (pp. 2 of 5)

4. Draw another right triangle with legs of three units using the same right angle as before. Use the value of the hypotenuse in the first triangle to determine the length of the hypotenuse in this triangle.

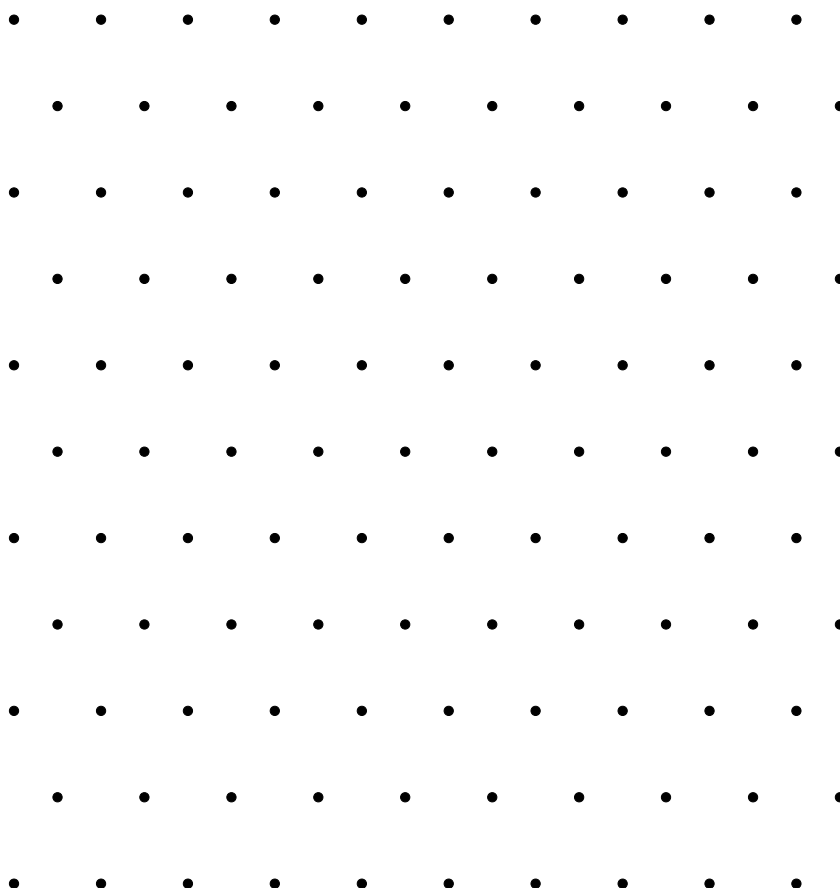
45°-45°-90° Triangle			
Length of 1st leg	Length of 2 <sup>nd</sup> leg	Length of Hypotenuse	Ratio of 1 <sup>st</sup> leg to 2 <sup>nd</sup> leg to hypotenuse
1	1		1 : 1: _____
2	2		
3	3		

5. Based on your data in the table above, write a conjecture about the length of the hypotenuse given the length of a leg of a 45°-45°-90° triangle.
6. Test your conjecture. If an isosceles right triangle has legs equal to eight units, predict the hypotenuse. Check your prediction by drawing the figure on the grid.
7. Suppose each leg of a 45°-45°-90° Triangle has a length of  $x$ . Express the ratio of the lengths of the sides of the triangle in the form *1<sup>st</sup> leg: 2<sup>nd</sup> leg: hypotenuse*.
8. Use your conjecture from question 5 and your ratio above to find the side lengths of a 45°-45°-90° Triangle given a leg length of 25.

## Exploring Special Right Triangles (pp. 3 of 5)

### 30°-60°-90° Triangles

Below is an isometric grid. The points form equilateral triangles; therefore the horizontal and diagonal distances between points are equal.



9. Let the lower left corner of the grid be the vertex of a right angle. Draw a right triangle so that the vertical leg extends to the first point above the vertex of the right angle and so that the horizontal leg extends to the first point to the right of the vertex of the right angle. Since the dots of the isometric grid form equilateral triangles, what is the measure of one of the acute angles of the right triangle in your drawing? What is the measure of the other acute angle? Explain your reasoning.

## Exploring Special Right Triangles (pp. 4 of 5)

10. Since the horizontal and diagonal distances between the points are equal, how long is the short leg of your right triangle? How long is the hypotenuse? Use this information to find the longer leg of the right triangle (express your answer as a radical). How did you find your answer?
11. Extend each leg of your right triangle to the next point on the grid paper and draw the hypotenuse of another right triangle. How long is the hypotenuse? Use the value of the length of the long leg from the first triangle to determine the length of the long leg in the second triangle.
12. Repeat the above procedure by extending the legs of the triangle to the next point on the grid paper. Find the lengths of the sides of the triangle. Record your answers in the table below.

30°-60°-90° Triangle			
Length of short leg	Length of Hypotenuse	Length of long leg	Ratio of short leg to hypotenuse to long leg
1	2		1: 2: _____
2	4		
3			

13. Based on your data in the table above, write a conjecture about the length of the hypotenuse and the length of the long leg given the length of the short leg of a 30°-60°-90° Triangle.



## Special Right Triangles (pp. 1 of 2) **KEY**

**Theorem:** In a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle, the legs are equal in measure and the hypotenuse is  $\sqrt{2}$  times as long as a leg.

1. Find the missing sides of the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangles represented in the table below.

Side	Side	Hypotenuse
7	7	$7\sqrt{2}$
24	24	$24\sqrt{2}$
$24\sqrt{2}$	$24\sqrt{2}$	48
$6\sqrt{2}$	$6\sqrt{2}$	12

2. The perimeter of a square is 36. What is the length of a side of the square? The length of the diagonal?

Side length is 9 units. Diagonal length is  $9\sqrt{2}$  units.

3. Suburban Susie has a backyard that is a square with sides of 100 feet. She wants to have a fence built with one diagonal to separate her yard into two parts. (One for her pool, one for the children's playground.) How many feet of fencing will she need to purchase (total feet needed to fence 3 sides of her yard and install the diagonal). Round answer to the nearest tenth of a foot.

441.4 ft.

## Special Right Triangles (pp. 2 of 2) **KEY**

**Theorem:** In a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is  $\sqrt{3}$  times as long as the shorter leg.

1. Find the missing sides of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles represented in the table below.

Short Side	Long Side	Hypotenuse
5	$5\sqrt{3}$	10
12	$12\sqrt{3}$	24
10	$10\sqrt{3}$	20
$8\sqrt{3}$	24	$16\sqrt{3}$
$18\sqrt{3}$	54	$36\sqrt{3}$
$15\sqrt{3}$	45	$30\sqrt{3}$

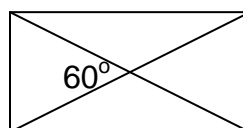
2. The altitude of an equilateral triangle is 12 cm. Find the perimeter of the equilateral triangle.

$24\sqrt{3}$  cm

3. The diagonals of rectangles bisect one another. In the rectangle below, the diagonals are 60 feet long and intersect at an angle of  $60^\circ$  as shown in the figure. Find the perimeter of the rectangle.

$(60 + 60\sqrt{3})$  ft.

163.9 ft.



4. The bed of a gravel truck is lifted to pour out a load of rock. If the bed of the truck is 12 feet long, how high is it from the frame when it is tipped upward at a  $30^\circ$  angle?

6 ft.

## Special Right Triangles (pp. 1 of 2)

**Theorem:** In a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle, the legs are equal in measure and the hypotenuse is  $\sqrt{2}$  times as long as a leg.

1. Find the missing sides of the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangles represented in the table below.

Side	Side	Hypotenuse
7		
		$24\sqrt{2}$
	$24\sqrt{2}$	
		12

2. The perimeter of a square is 36. What is the length of a side of the square? The length of the diagonal?
3. Suburban Susie has a backyard that is a square with sides of 100 feet. She wants to have a fence built with one diagonal to separate her yard into two parts. (One for her pool, one for the children's playground.) How many feet of fencing will she need to purchase (total feet needed to fence 3 sides of her yard and install the diagonal). Round answer to the nearest tenth of a foot.

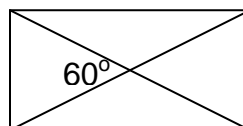
## Special Right Triangles (pp. 2 of 2)

**Theorem:** In a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is  $\sqrt{3}$  times as long as the shorter leg.

1. Find the missing sides of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles represented in the table below.

Short Side	Long Side	Hypotenuse
5		
	$12\sqrt{3}$	
		20
	24	
		$36\sqrt{3}$
$15\sqrt{3}$		

2. The altitude of an equilateral triangle is 12 cm. Find the perimeter of the equilateral triangle.
3. The diagonals of rectangles bisect one another. In the rectangle below, the diagonals are 60 feet long and intersect at an angle of  $60^\circ$  as shown in the figure. Find the perimeter of the rectangle.

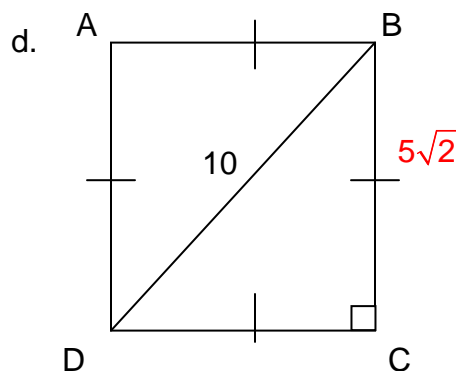
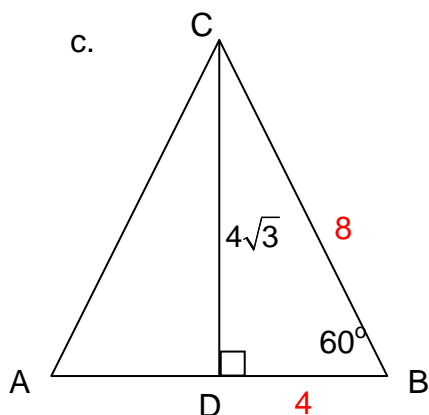
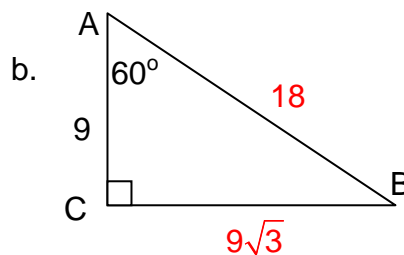
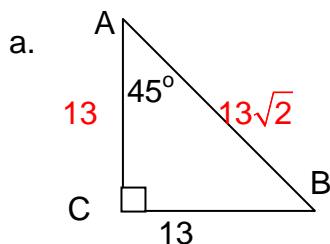


4. The bed of a gravel truck is lifted to pour out a load of rock. If the bed of the truck is 12 feet long, how high is it from the frame when it is tipped upward at a  $30^\circ$  angle?



## Applications of Special Triangles (pp. 1 of 2) **KEY**

1. Use the special right triangle relationships to find the measures of the values of x and y. Leave answers in radical form.



2. Farmer Fred had a square field with sides of 525 feet. He wants to build a fence down one diagonal of the field to separate it into two equal parts. How many feet of fencing will he need to purchase? Show two methods to find the solution.

By  $45^\circ$ - $45^\circ$ - $90^\circ$  Theorem:

525: 525:  $525\sqrt{2}$  therefore  $525\sqrt{2}$  or 742.5 ft.

Pythagorean Theorem:

$$d^2 = 525^2 + 525^2$$

Therefore,  $d = 525\sqrt{2}$  or 742.5 ft.

3. The altitude of an equilateral triangle is 12 feet. Find the perimeter and area of the equilateral triangle.

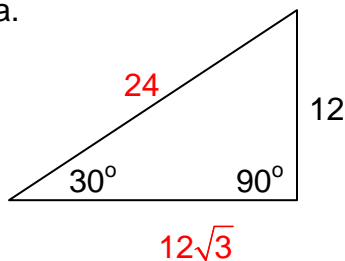
Perimeter:  $24\sqrt{3}$  ft. or 41.6 ft.

Area:  $48\sqrt{3}$  ft<sup>2</sup> or 83.1 ft<sup>2</sup>.

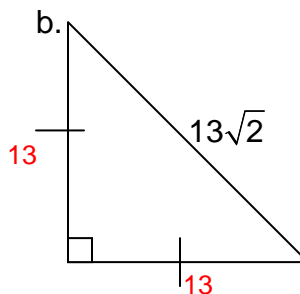
## Applications of Special Triangles (pp. 2 of 2) **KEY**

4. Use the special right triangle relationships to find the measures of the values of  $x$  and  $y$ . Leave answers in radical form.

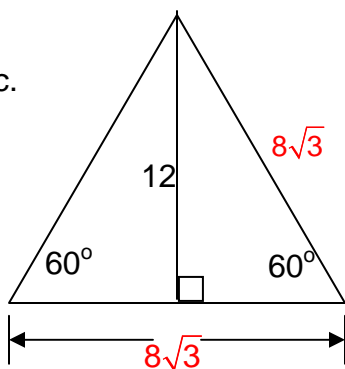
a.



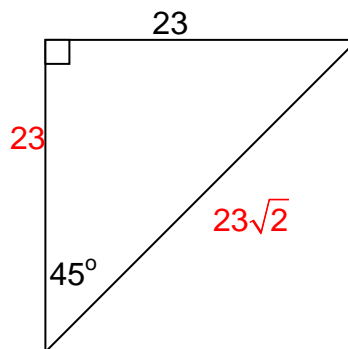
b.



c.



d.

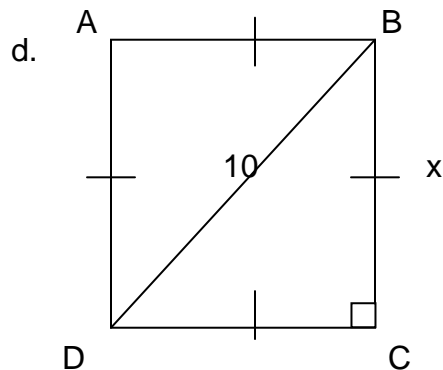
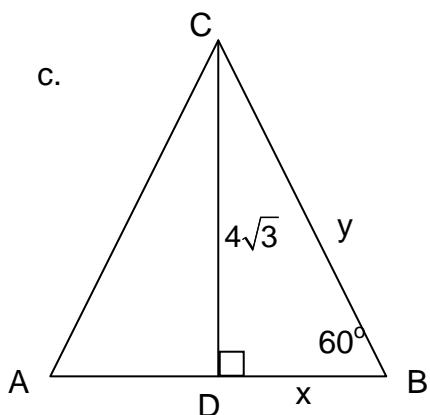
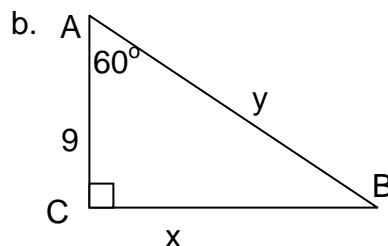
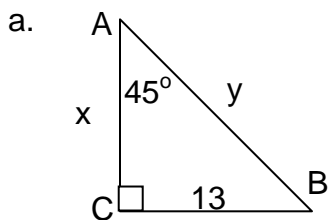


Round the following answers to the nearest hundredth.

5. The length of a diagonal of a square is 12 inches. Find the length of one side of the square.  
**8.49 in.**
6. The perimeter of an equilateral triangle is 39 centimeters. Find the length of an altitude of the triangle.  
**11.26 in.**
7. The diagonals of a rectangle are 15 inches long and bisect each other at an angle of  $60^\circ$ . Find the perimeter of the rectangle.  
**40.98 in.**

## Applications of Special Triangles (pp. 1 of 2)

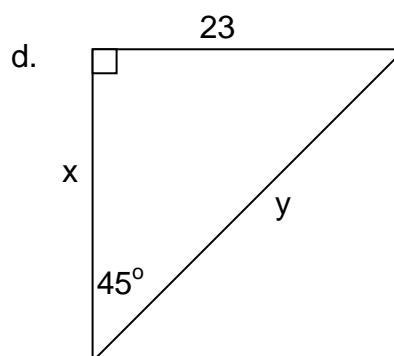
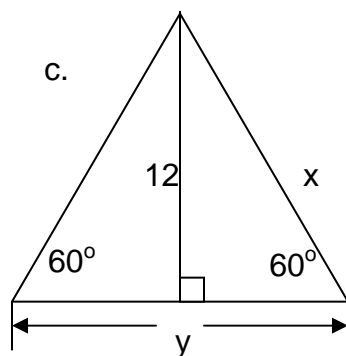
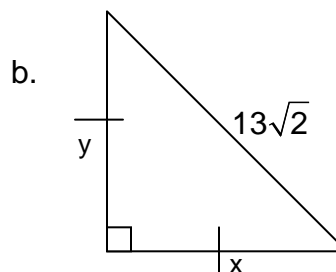
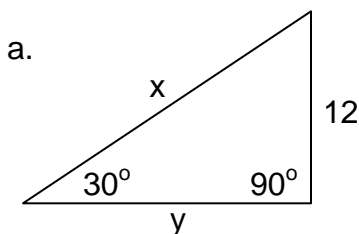
1. Use the special right triangle relationships to find the measures of the values of  $x$  and  $y$ . Leave answers in radical form.



2. Farmer Fred had a square field with sides of 525 feet. He wants to build a fence down one diagonal of the field to separate it into two equal parts. How many feet of fencing will he need to purchase? Show two methods to find the solution.
3. The altitude of an equilateral triangle is 12 feet. Find the perimeter and area of the equilateral triangle.

## Applications of Special Triangles (pp. 2 of 2)

4. Use the special right triangle relationships to find the measures of the values of  $x$  and  $y$ . Leave answers in radical form.



Round the following answers to the nearest hundredth.

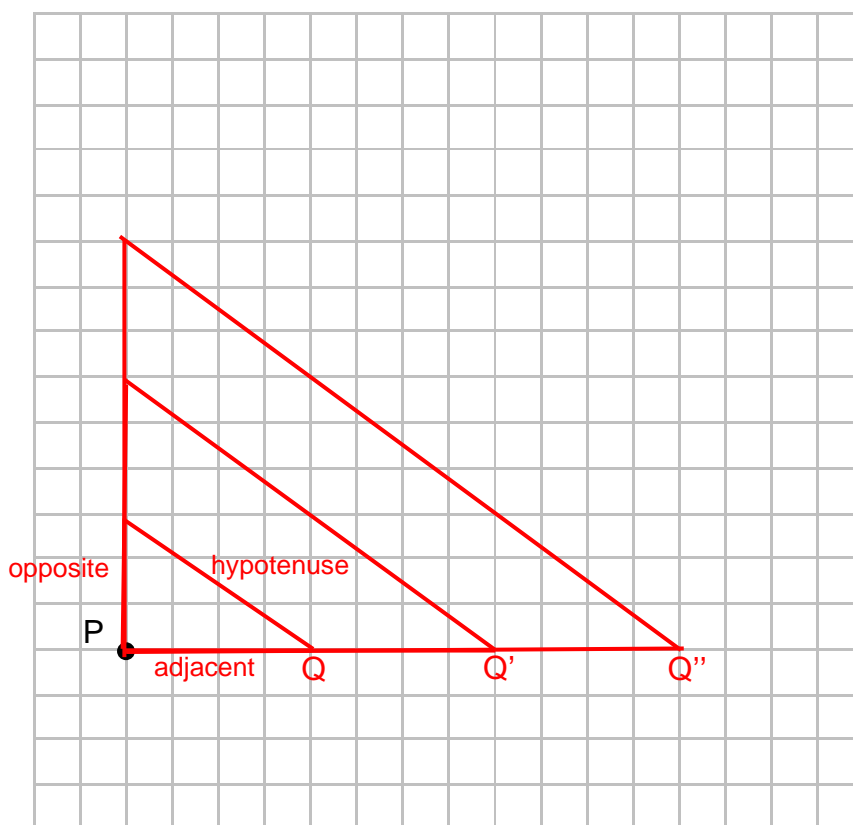
- The length of a diagonal of a square is 12 inches. Find the length of one side of the square.
- The perimeter of an equilateral triangle is 39 centimeters. Find the length of an altitude of the triangle.
- The diagonals of a rectangle are 15 inches long and bisect each other at an angle of  $60^\circ$ . Find the perimeter of the rectangle.

## Exploring Trigonometric Ratios (pp. 1 of 8) **KEY**

### Part I: Finding Ratios

In Part I, you will investigate relationships between angle measures and the ratio of sides of right triangles using your previous knowledge of right triangles.

- Using the grid below and P as a starting point, draw a right triangle so that the horizontal leg is 4 units long, the vertical leg is 3 units long, and  $\angle P$  is the right angle. Label the vertex of the acute angle to the right of P as Q. Use your knowledge of right triangles to calculate the length of the hypotenuse. Explain how you found the length of the hypotenuse.



- Label the horizontal leg of the triangle above *adjacent* (this leg is next to point Q); label the vertical leg of the triangle above *opposite* (this leg is across from point Q); and label the longest side *hypotenuse*. Use point P as the center of dilation to dilate the triangle by a scale factor of 2 to create a second triangle, and then by a scale factor of 3 to create a third triangle. Find the side lengths of the second and third triangles. Record your data in the table below.

Triangle	Opposite	Adjacent	Hypotenuse
1	3	4	5
2	6	8	10
3	9	12	15

## Exploring Trigonometric Ratios (pp. 2 of 8) **KEY**

3. What can you conclude about the three triangles that you drew on the grid? Explain your reasoning.

They are similar. The ratios of corresponding sides are equal.

4. Recall from the beginning of this lesson, carpenters express the angle the roof makes with the horizontal as a ratio of two sides of a right triangle. The acute angles of a right triangle can be expressed as the ratio of two sides of the triangle using Trigonometric ratios. The ratios are formed as follows:

- The ratio  $\frac{\text{opposite}}{\text{adjacent}}$  is called the *tangent* ratio (tan).
- The ratio  $\frac{\text{opposite}}{\text{hypotenuse}}$  is called the *sine* ratio (sin).
- The ratio  $\frac{\text{adjacent}}{\text{hypotenuse}}$  is called the *cosine* ratio (cos).

Use the table below to form the sine, cosine, and tangent ratios for  $\angle Q$  in each of the three triangles.

Triangle	$\sin \angle Q = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos \angle Q = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan \angle Q = \frac{\text{opposite}}{\text{adjacent}}$
1	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$
2	$\frac{6}{10} = \frac{3}{5}$	$\frac{8}{10} = \frac{4}{5}$	$\frac{6}{8} = \frac{3}{4}$
3	$\frac{9}{15} = \frac{3}{5}$	$\frac{12}{15} = \frac{4}{5}$	$\frac{9}{12} = \frac{3}{4}$

5. How do the sine ratios for  $\angle Q$  compare in each right triangle? Why do you suppose this is true?

They are equal, because the three triangles are similar.

6. How do the cosine ratios for  $\angle Q$  compare in each right triangle? Why do you suppose this is true?

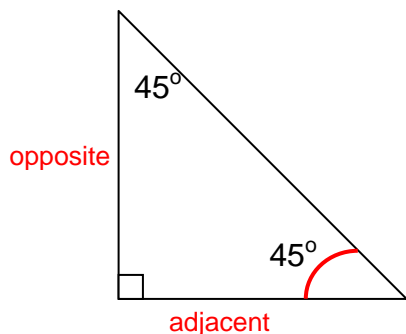
They are equal, because the three triangles are similar.

7. How do the tangent ratios for  $\angle Q$  compare in each right triangle? Why do you suppose this is true?

They are equal, because the three triangles are similar.

## Exploring Trigonometric Ratios (pp. 3 of 8) **KEY**

8. For the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle pictured below, label one of the legs *adjacent* and the other leg *opposite*. Complete the table based on your knowledge of  $45^\circ$ - $45^\circ$ - $90^\circ$  triangles.



45°-45°-90° Triangle		
Opposite	Adjacent	Hypotenuse
1	1	$\sqrt{2}$

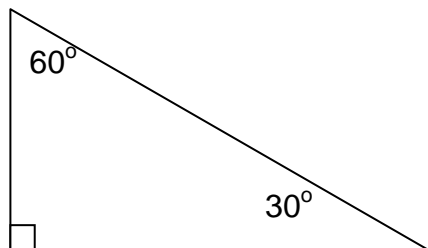
Use the information above to find the sine, cosine, and tangent ratios for  $45^\circ$ .

$\sin 45^\circ$	$\cos 45^\circ$	$\tan 45^\circ$
$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{1}$

9. Suppose the leg length of the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle above was changed to another value. Would the sine, cosine, and tangent ratios for  $45^\circ$  change? Explain your reasoning.

**No, since the triangles would be similar and therefore the corresponding angles equal, the trig ratios would not change.**

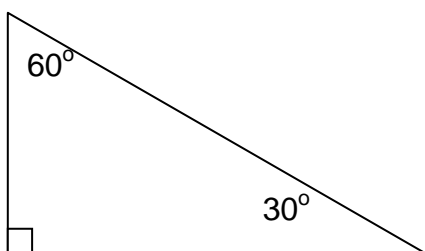
Use the table below for the following questions.



30°-60°-90° Triangle		
Short Leg	Long Leg	Hypotenuse
1	$\sqrt{3}$	2

## Exploring Trigonometric Ratios (pp. 4 of 8) **KEY**

10. If the short leg of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle is one unit long, use your knowledge of  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles to find the other side lengths. Record your data in the table beside the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle above.
- For the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, *opposite* and *adjacent* are relative terms. Which side (short leg or long leg) is next to or *adjacent* to the  $30^\circ$  angle?  
**long leg**
  - Which side is across from or *opposite* the  $30^\circ$  angle?  
**short leg**
  - Which side (short leg or long leg) is next to or *adjacent* the  $60^\circ$  angle?  
**short leg**
  - Which side is across from or *opposite* the  $60^\circ$  angle?  
**long leg**
  - Use your answers to the questions above to find the sine, cosine, and tangent ratios for  $30^\circ$  and for  $60^\circ$ . Complete the table below.



Use the information above to find the sine, cosine, and tangent ratios for  $30^\circ$  and for  $60^\circ$ .

$\sin 30^\circ$	$\cos 30^\circ$	$\tan 30^\circ$
$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\sin 60^\circ$	$\cos 60^\circ$	$\tan 60^\circ$
$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1}$

11. Suppose the short leg length of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle above was changed to another value. Would the sine, cosine, and tangent ratios for  $30^\circ$  and  $60^\circ$  change? Explain your reasoning.  
**No, since the triangles would be similar and therefore the corresponding angles equal, the trig ratios would not change.**



## Exploring Trigonometric Ratios (pp. 5 of 8) **KEY**

### Part II: Using the Calculator

In Part II, you will investigate the role the calculator plays in using trigonometric ratios.

In Part I: Finding Ratios, you found the sine, cosine, and tangent ratios given the side lengths of a triangle. You also discovered that the trigonometric ratios for a particular angle did not depend on any one side length of a triangle but rather a *ratio* of side lengths. For example, in question 4 the sine ratios for  $\angle P$  were the same for each of the three similar triangles. The same was true for the cosine and tangent ratios for  $\angle P$ . In fact, the values for the trigonometric ratios have been calculated and stored in your calculator.

12. To use your graphing calculator to find trigonometric ratios, you must first set your calculator to *degree* mode. For the TI series of calculators, press MODE and on the third line of the menu, use your cursor KEYS to highlight DEGREE and press ENTER. Press 2<sup>ND</sup> and MODE to return to the previous screen. Your calculator can now be used to find trigonometric ratios. To find the sine of  $45^\circ$ , press SIN, then 4, then 5, and then ENTER. Your calculator should display .7071067812. Use the graphing calculator to complete the table below.

Angle	Sine	Cosine	Tangent
$30^\circ$	0.5	0.8660	0.5774
$45^\circ$	0.7071	0.7071	1
$60^\circ$	0.8660	0.5	1.7321

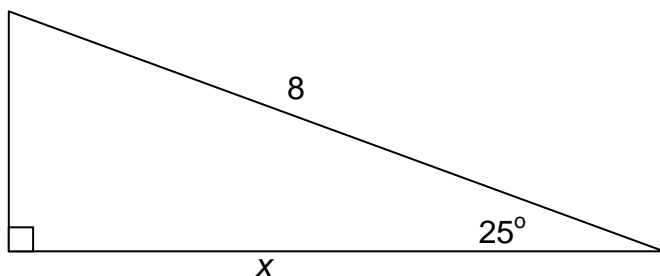
13. Compare the calculator's trigonometric ratios in the table above with those that you found based on the side lengths of triangles. How do they compare? Is this what you would expect? Why or why not?

The calculator's ratios are decimal equivalents of the ratios based on side lengths.

14. The calculator can be used to find a particular trigonometric ratio given any acute angle of a right triangle. Find the sine, cosine, and tangent ratios for  $25^\circ$ , and record your answers below.  
 $\sin 25^\circ = .4226$ ;  $\cos 25^\circ = .9063$ ;  $\tan 25^\circ = .4663$

## Exploring Trigonometric Ratios (pp. 6 of 8) **KEY**

15. Consider the right triangle below.



- What is the length of the side adjacent to  $25^\circ$ ?  
**x**
- What is the length of the hypotenuse?  
**8**
- Which trigonometric ratio of  $25^\circ$  can be formed using the values given in parts a and b?  
**cosine ratio**
- Write a trigonometric ratio for  $25^\circ$  in terms of  $x$  and 8.

$$\cos 25^\circ = \frac{x}{8}$$

- In question 14, you used the calculator to find values for the trigonometric ratios for  $25^\circ$ . Use the appropriate value from question 14 and your answer in part d to write an equation that can be solved for  $x$ . Find the value of  $x$ .

$$.9063 = \frac{x}{8}$$

$$x = 7.25$$

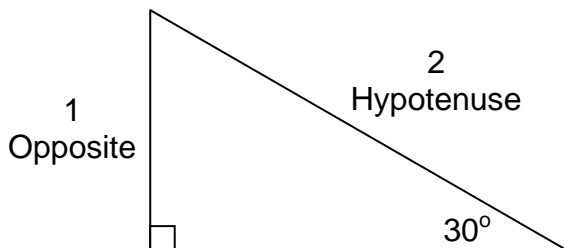
- Label the other leg of the right triangle  $y$ . Use a similar procedure to solve for  $y$ . Show your work below.

$$\sin 25^\circ = \frac{y}{8}$$

$$y = 3.38$$

## Exploring Trigonometric Ratios (pp. 7 of 8) **KEY**

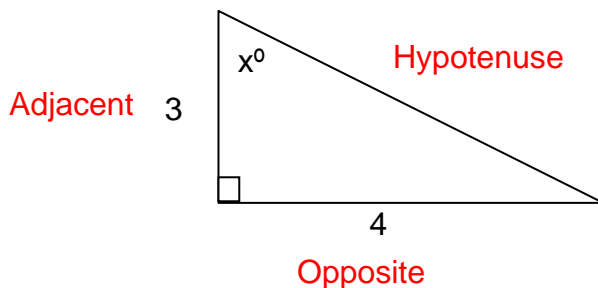
16. The calculator can also find the angle value given a particular trigonometric ratio. Consider the triangle below.



- In the triangle above,  $\sin 30^\circ$  is the ratio  $\frac{\text{opposite}}{\text{hypotenuse}}$  or  $\sin 30^\circ = \frac{1}{2}$ .
  - To let the calculator find the angle that has a sin ratio of  $\frac{1}{2}$ , enter the following: 2<sup>nd</sup>, SIN, 1,  $\div$ , 2, and then ENTER. (Make sure the MODE on the calculator is in degrees and not radians.)
  - Your calculator should display 30.
  - The notation for this operation is written  $\sin^{-1}(1/2) = x^\circ$  or  $\sin^{-1}(1/2) = 30^\circ$ .
  - It is read as “The inverse sine of  $\frac{1}{2}$  is some angle  $x$ ,” or “The inverse sin of  $\frac{1}{2}$  is  $30^\circ$ .”
- What is the measure of the other acute angle? How do you know?  
 **$60^\circ$  by Triangle Angle Sum Theorem**
  - What is the length of the other leg (adjacent)? How do you know?  
 **$\sqrt{3}$ , by  $30^\circ$ - $60^\circ$ - $90^\circ$  Triangle Theorem**
  - Use a different trigonometric ratio and your calculator to verify the  $30^\circ$  angle. Write your notation for this operation below.  
 **$\cos^{-1}(\sqrt{3}/2) = x^\circ$**

## Exploring Trigonometric Ratios (pp. 8 of 8) **KEY**

17. Consider the right triangle below.



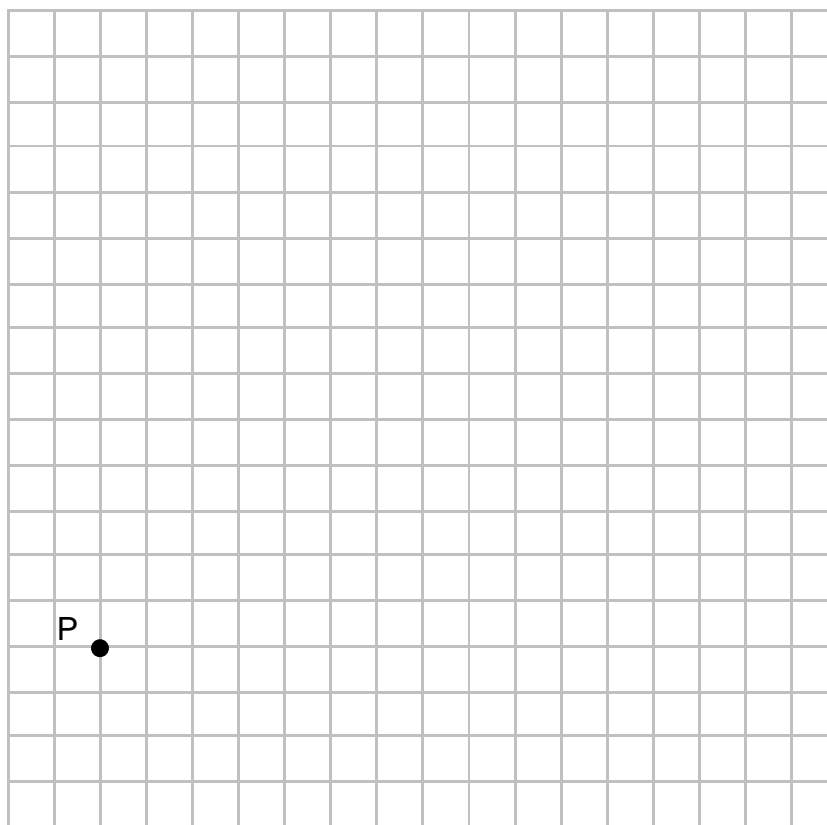
- For the angle  $x^\circ$ , label the opposite, adjacent, and hypotenuse sides of the triangle above.
- For the angle  $x^\circ$ , which trigonometric ratio can be formed with the 3 and the 4?  
**tangent**
- Write the equation notation that would allow you to find angle  $x^\circ$ .  
 **$\tan^{-1}(4/3) = x^\circ$**
- Use the notation you wrote in part c and your calculator to find the value of  $x^\circ$ .  
 **$53.1^\circ$**
- What is the length of the hypotenuse? How do you know?  
**5, by the Pythagorean Theorem**
- Use the inverse of another trigonometric ratio to verify the value of  $x^\circ$ . Write the notation below.  
 **$\sin^{-1}(4/5) = x^\circ$  or  $\cos^{-1}(3/5) = x^\circ$**

## Exploring Trigonometric Ratios (pp. 1 of 8)

### Part I: Finding Ratios

In Part I, you will investigate relationships between angle measures and the ratio of sides of right triangles using your previous knowledge of right triangles.

- Using the grid below and P as a starting point, draw a right triangle so that the horizontal leg is 4 units long, the vertical leg is 3 units long, and  $\angle P$  is the right angle. Label the vertex of the acute angle to the right of P as Q. Use your knowledge of right triangles to calculate the length of the hypotenuse. Explain how you found the length of the hypotenuse.



- Label the horizontal leg of the triangle above *adjacent* (this leg is next to point Q); label the vertical leg of the triangle above *opposite* (this leg is across from point Q); and label the longest side *hypotenuse*. Use point P as the center of dilation to dilate the triangle by a scale factor of 2 to create a second triangle and then by a scale factor of 3 to create a third triangle. Find the side lengths of the second and third triangles. Record your data in the table below.

Triangle	Opposite	Adjacent	Hypotenuse
1	3	4	
2			
3			

## Exploring Trigonometric Ratios (pp. 2 of 8)

3. What can you conclude about the three triangles that you drew on the grid? Explain your reasoning.
4. Recall from the beginning of this lesson, carpenters express the angle the roof makes with the horizontal as a ratio of two sides of a right triangle. The acute angles of a right triangle can be expressed as the ratio of two sides of the triangle using Trigonometric ratios. The ratios are formed as follows:

- The ratio  $\frac{\text{opposite}}{\text{adjacent}}$  is called the *tangent* ratio (tan).
- The ratio  $\frac{\text{opposite}}{\text{hypotenuse}}$  is called the *sine* ratio (sin).
- The ratio  $\frac{\text{adjacent}}{\text{hypotenuse}}$  is called the *cosine* ratio (cos).

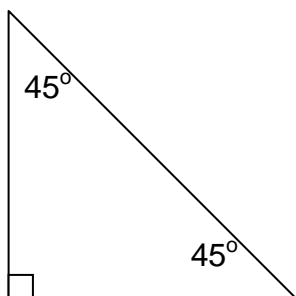
Use the table below to form the sine, cosine, and tangent ratios for  $\angle P$  in each of the three triangles.

Triangle	$\sin \angle Q = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos \angle Q = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan \angle Q = \frac{\text{opposite}}{\text{adjacent}}$
1			
2			
3			

5. How do the sine ratios for  $\angle Q$  compare in each right triangle? Why do you suppose this is true?
6. How do the cosine ratios for  $\angle Q$  compare in each right triangle? Why do you suppose this is true?
7. How do the tangent ratios for  $\angle Q$  compare in each right triangle? Why do you suppose this is true?

## Exploring Trigonometric Ratios (pp. 3 of 8)

8. For the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle pictured below, label one of the legs *adjacent*, and the other leg *opposite*. Complete the table based on your knowledge of  $45^\circ$ - $45^\circ$ - $90^\circ$  triangles.



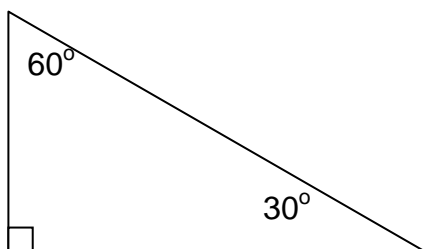
<b><math>45^\circ</math>-<math>45^\circ</math>-<math>90^\circ</math> Triangle</b>		
<b>Opposite</b>	<b>Adjacent</b>	<b>Hypotenuse</b>
1		

Use the information above to find the sine, cosine, and tangent ratios for  $45^\circ$ .

<b><math>\sin 45^\circ</math></b>	<b><math>\cos 45^\circ</math></b>	<b><math>\tan 45^\circ</math></b>

9. Suppose the leg length of the  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle above was changed to another value. Would the sine, cosine, and tangent ratios for  $45^\circ$  change? Explain your reasoning.

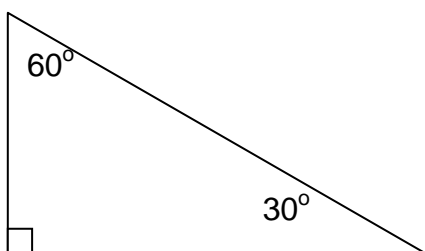
Use the table below for the following questions.



<b><math>30^\circ</math>-<math>60^\circ</math>-<math>90^\circ</math> Triangle</b>		
<b>Short Leg</b>	<b>Long Leg</b>	<b>Hypotenuse</b>
1		

## Exploring Trigonometric Ratios (pp. 4 of 8)

10. If the short leg of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle is one unit long, use your knowledge of  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles to find the other side lengths. Record your data in the table beside the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle above.
- For the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, *opposite* and *adjacent* are relative terms. Which side (short leg or long leg) is next to or *adjacent* to the  $30^\circ$  angle?
  - Which side is across from or *opposite* the  $30^\circ$  angle?
  - Which side (short leg or long leg) is next to or *adjacent* the  $60^\circ$  angle?
  - Which side is across from or *opposite* the  $60^\circ$  angle?
  - Use your answers to the questions above to find the sine, cosine, and tangent ratios for  $30^\circ$  and for  $60^\circ$ . Complete the table below.



Use the information above to find the sine, cosine, and tangent ratios for  $30^\circ$  and for  $60^\circ$ .

$\sin 30^\circ$	$\cos 30^\circ$	$\tan 30^\circ$
$\sin 60^\circ$	$\cos 60^\circ$	$\tan 60^\circ$

11. Suppose the short leg length of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle above was changed to another value. Would the sine, cosine, and tangent ratios for  $30^\circ$  and  $60^\circ$  change? Explain your reasoning.



## Exploring Trigonometric Ratios (pp. 5 of 8)

### Part II: Using the Calculator

In Part II, you will investigate the role the calculator plays in using trigonometric ratios.

In Part I: Finding Ratios, you found the sine, cosine, and tangent ratios given the side lengths of a triangle. You also discovered that the trigonometric ratios for a particular angle did not depend on any one side length of a triangle but rather a *ratio* of side lengths. For example, in question 4 the sine ratios for  $\angle P$  were the same for each of the three similar triangles. The same was true for the cosine and tangent ratios for  $\angle P$ . In fact, the values for the trigonometric ratios have been calculated and stored in your calculator.

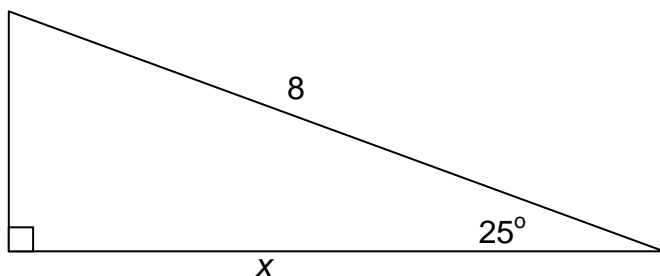
12. To use your graphing calculator to find trigonometric ratios, you must first set your calculator to *degree* mode. For the TI series of calculators, press MODE and on the third line of the menu, use your cursor KEYS to highlight DEGREE and press ENTER. Press 2<sup>ND</sup> and MODE to return to the previous screen. Your calculator can now be used to find trigonometric ratios. To find the sine of  $45^\circ$ , press SIN, then 4, then 5, and then ENTER. Your calculator should display .7071067812. Use the graphing calculator to complete the table below.

Angle	Sine	Cosine	Tangent
$30^\circ$			
$45^\circ$			
$60^\circ$			

13. Compare the calculator's trigonometric ratios in the table above with those that you found based on the side lengths of triangles. How do they compare? Is this what you would expect? Why or why not?
  
14. The calculator can be used to find a particular trigonometric ratio given any acute angle of a right triangle. Find the sine, cosine, and tangent ratios for  $25^\circ$  and record your answers below.

## Exploring Trigonometric Ratios (pp. 6 of 8)

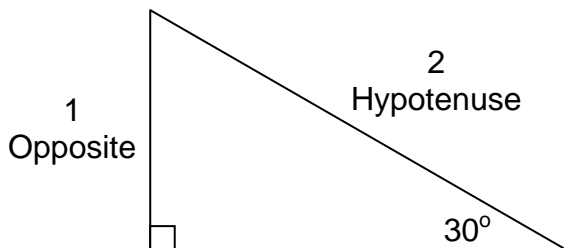
15. Consider the right triangle below.



- What is the length of the side adjacent to  $25^\circ$ ?
- What is the length of the hypotenuse?
- Which trigonometric ratio of  $25^\circ$  can be formed using the values given in parts a and b?
- Write a trigonometric ratio for  $25^\circ$  in terms of  $x$  and 8.
- In question 14, you used the calculator to find values for the trigonometric ratios for  $25^\circ$ . Use the appropriate value from question 14 and your answer in part d to write an equation that can be solved for  $x$ . Find the value of  $x$ .
- Label the other leg of the right triangle  $y$ . Use a similar procedure to solve for  $y$ . Show your work below.

## Exploring Trigonometric Ratios (pp. 7 of 8)

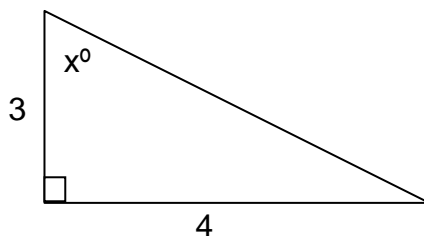
16. The calculator can also find the angle value given a particular trigonometric ratio. Consider the triangle below.



- In the triangle above,  $\sin 30^\circ$  is the ratio  $\frac{\text{opposite}}{\text{hypotenuse}}$  or  $\sin 30^\circ = \frac{1}{2}$ .
  - To let the calculator find the angle that has a sin ratio of  $\frac{1}{2}$ , enter the following: 2<sup>nd</sup>, SIN, 1, ÷, 2, and then ENTER. (Make sure the MODE on the calculator is in degrees and not radians.)
  - Your calculator should display 30.
  - The notation for this operation is written  $\sin^{-1}(1/2) = x^\circ$  or  $\sin^{-1}(1/2) = 30^\circ$ .
  - It is read as “The inverse sine of  $\frac{1}{2}$  is some angle x,” or “The inverse sin of  $\frac{1}{2}$  is  $30^\circ$ .”
- a. What is the measure of the other acute angle? How do you know?
- b. What is the length of the other leg (adjacent)? How do you know?
- c. Use a different trigonometric ratio and your calculator to verify the  $30^\circ$  angle. Write your notation for this operation below.

## Exploring Trigonometric Ratios (pp. 8 of 8)

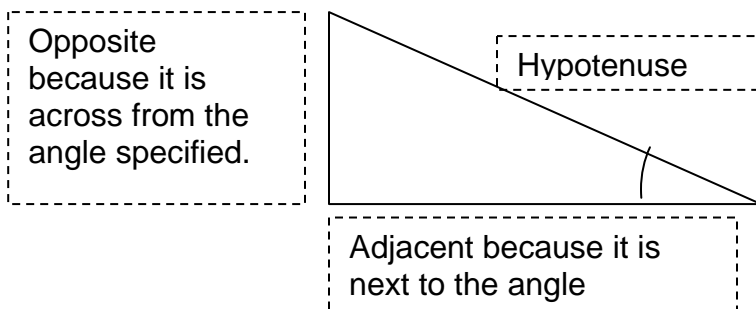
17. Consider the right triangle below.



- For the angle  $x^\circ$ , label the opposite, adjacent, and hypotenuse sides of the triangle above.
- For the angle  $x^\circ$ , which trigonometric ratio can be formed with the 3 and the 4?
- Write the equation notation that would allow you to find angle  $x^\circ$ .
- Use the notation you wrote in part c and your calculator to find the value of  $x^\circ$ .
- What is the length of the hypotenuse? How do you know?
- Use the inverse of another trigonometric ratio to verify the value of  $x^\circ$ . Write the notation below.

## Trigonometric Ratios (pp. 1 of 4) **KEY**

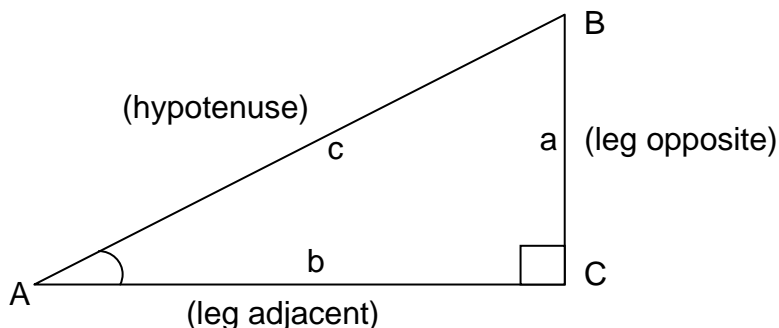
The word trigonometry is derived from the Latin words for triangle (*trigon*) and measurement (*metry*). Trigonometry is the study of the relationship between the angles and sides of triangles. Although this lesson will concentrate on right triangles, trigonometry can be applied to all triangles.



The ratio  $\frac{\text{opposite}}{\text{adjacent}}$  is called the tangent ratio (tan).

The ratio  $\frac{\text{opposite}}{\text{hypotenuse}}$  is called the sine ratio (sin).

The ratio  $\frac{\text{adjacent}}{\text{hypotenuse}}$  is called the cosine ratio (cos).



$$\sin \angle A = \frac{a}{c}$$

$$\cos \angle A = \frac{b}{c}$$

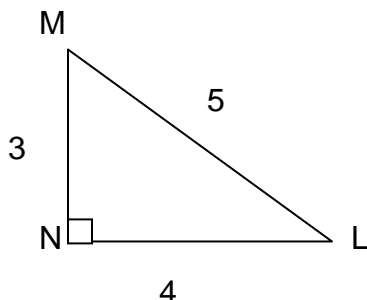
$$\tan \angle A = \frac{a}{b}$$

## Trigonometric Ratios (pp. 2 of 4) **KEY**

### EXAMPLES

The value of trigonometric ratios can be found using given side measures of a right triangle.

- Find the  $\sin \angle L$ ,  $\cos \angle L$ ,  $\tan \angle L$ ,  $\sin \angle M$ ,  $\cos \angle M$ , and  $\tan \angle M$ . Express answers in fraction and decimal form. Round decimals to the nearest hundredth.



$\sin \angle L = \frac{3}{5} = 0.60$	$\sin \angle M = \frac{4}{5} = 0.80$
$\cos \angle L = \frac{4}{5} = 0.80$	$\cos \angle M = \frac{3}{5} = 0.60$
$\tan \angle L = \frac{3}{4} = 0.75$	$\tan \angle M = \frac{4}{3} = 1.33$

Given an acute angle of a right triangle, any of the trig functions can be found using tables or a calculator.

- Use a calculator to express each answer accurate to ten thousandths.
 

a. $\sin 42^\circ = x$	b. $\cos 62^\circ = x$	c. $\tan 72^\circ = x$
<b>0.6691</b>	<b>0.4695</b>	<b>3.0777</b>

Given side measures of a right triangle, either of the two acute angles can be found using tables or a calculator.

- Use a calculator to find the angles to the nearest tenth of a degree.
 

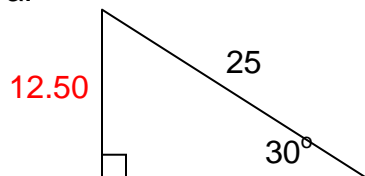
a. $\sin A = 0.7245$	b. $\cos F = 0.1212$	c. $\tan M = 0.4279$
<b>46.4°</b>	<b>83.0°</b>	<b>23.2°</b>

## Trigonometric Ratios (pp. 3 of 4) **KEY**

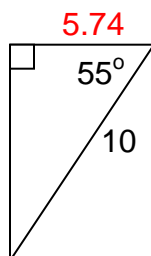
Trigonometric ratios can be used to solve problems involving right triangles.

4. Find the value of the variable in the following figures. Round answers to the nearest hundredth.

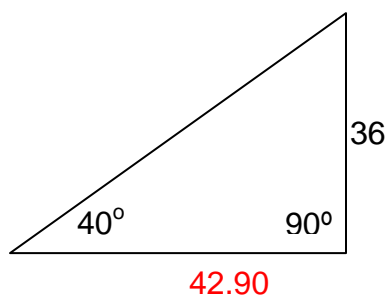
a.



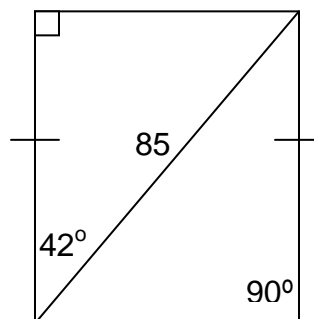
b.



c.



d.

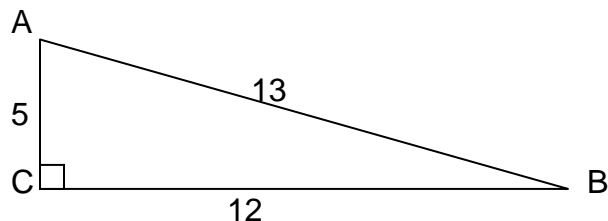


The congruent sides are also parallel.  
Perimeter of quadrilateral = 240.09

# Trigonometric Ratios (pp. 4 of 4) **KEY**

## Practice Problems

1. Find the trigonometric ratios for both acute angles in fraction and decimal form.



2. Find the value to the nearest ten-thousandth.

a.  $\tan 25^\circ = x$

**0.4663**

b.  $\sin 85^\circ = x$

**0.9962**

c.  $\cos 32^\circ = x$

**0.8480**

3. Find the measure of each angle to the nearest tenth of a degree.

a.  $\tan R = 9.4618$

**84.0°**

b.  $\sin S = 0.4567$

**27.2°**

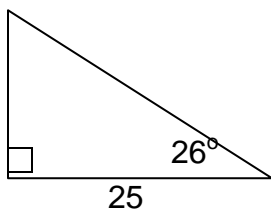
c.  $\cos T = 0.7431$

**42.0°**

4. Find the value of the variable in the following figures. Round answers to the nearest hundredth.

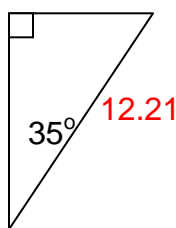
a.

**12.19**



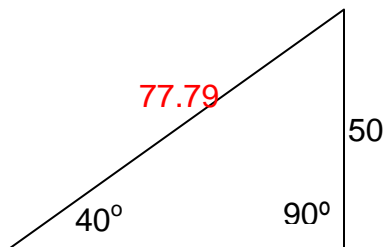
b.

10

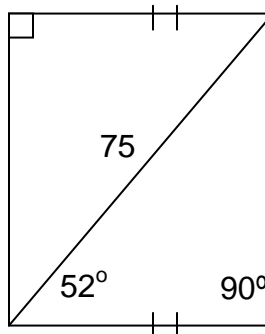


c.

**77.79**



d.

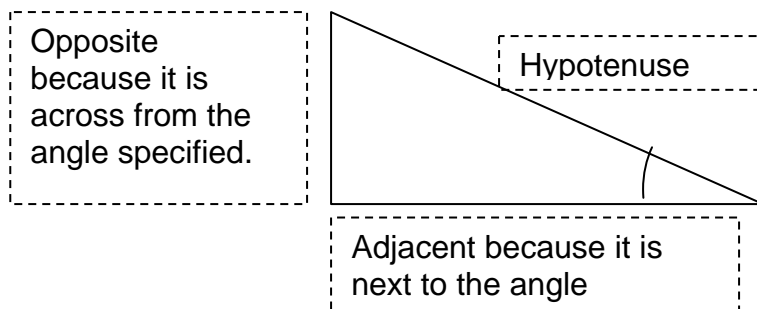


Congruent sides are also parallel.  
Perimeter of quadrilateral = **210.55**



## Trigonometric Ratios (pp. 1 of 4)

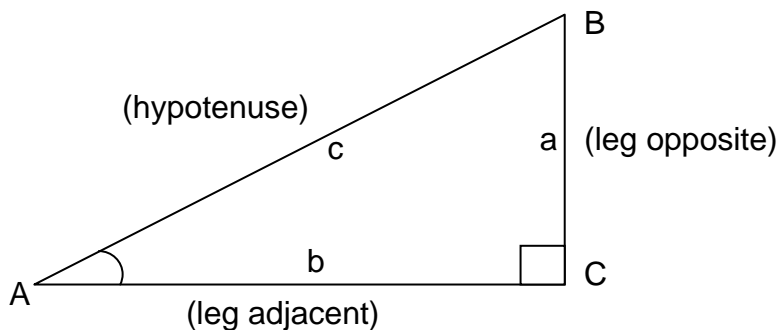
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The ratio  $\frac{\text{adjacent}}{\text{hypotenuse}}$  is called the cosine ratio (cos).



$$\sin \angle A = \frac{a}{c}$$

$$\cos \angle A = \frac{b}{c}$$

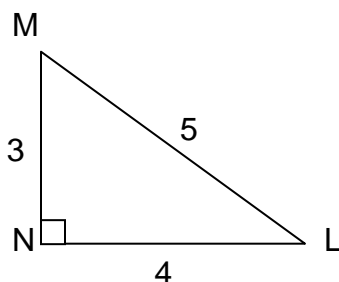
$$\tan \angle A = \frac{a}{b}$$

## Trigonometric Ratios (pp. 2 of 4)

### EXAMPLES:

The value of trigonometric ratios can be found using given side measures of a right triangle.

- Find the  $\sin \angle L$ ,  $\cos \angle L$ ,  $\tan \angle L$ ,  $\sin \angle M$ ,  $\cos \angle M$ , and  $\tan \angle M$ . Express answers in fraction and decimal form. Round decimals to the nearest hundredth.



$\sin \angle L =$	$\sin \angle M =$
$\cos \angle L =$	$\cos \angle M =$
$\tan \angle L =$	$\tan \angle M =$

Given an acute angle of a right triangle, any of the trig functions can be found using tables or a calculator.

- Use a calculator to express each answer accurate to ten thousandths.
  - $\sin 42^\circ = x$
  - $\cos 62^\circ = x$
  - $\tan 72^\circ = x$

Given side measures of a right triangle, either of the two acute angles can be found using tables or a calculator.

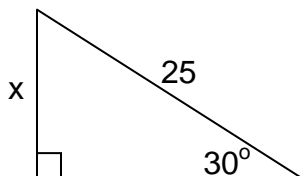
- Use a calculator to find the angles to the nearest tenth of a degree.
  - $\sin A = 0.7245$
  - $\cos F = 0.1212$
  - $\tan M = 0.4279$

## Trigonometric Ratios (pp. 3 of 4)

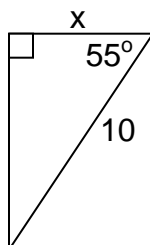
Trigonometric ratios can be used to solve problems involving right triangles.

4. Find the value of the variable in the following figures. Round answers to the nearest hundredth.

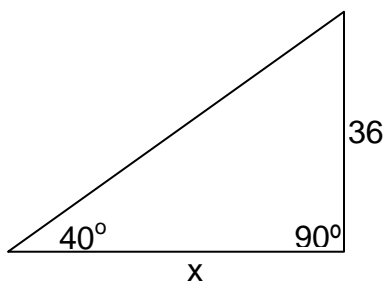
a.



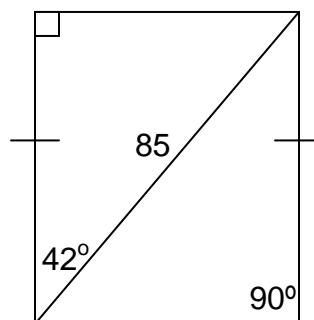
b.



c.



d.

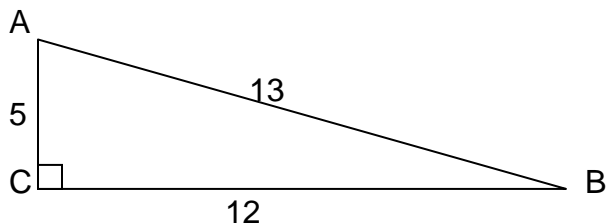


The congruent sides are also parallel.  
Perimeter of quadrilateral = \_\_\_\_\_

# Trigonometric Ratios (pp. 4 of 4)

## Practice Problems

1. Find the trigonometric ratios for both acute angles in fraction and decimal form.



2. Find the value to the nearest ten-thousandth.

a.  $\tan 25^\circ = x$

b.  $\sin 85^\circ = x$

c.  $\cos 32^\circ = x$

3. Find the measure of each angle to the nearest tenth of a degree.

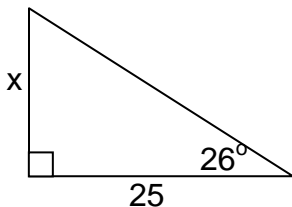
a.  $\tan R = 9.4618$

b.  $\sin S = 0.4567$

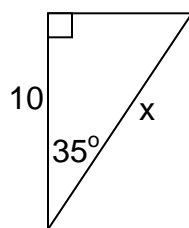
c.  $\cos T = 0.7431$

4. Find the value of the variable in the following figures. Round answers to the nearest hundredth.

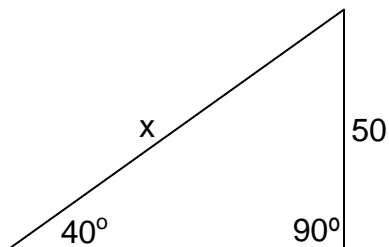
a.



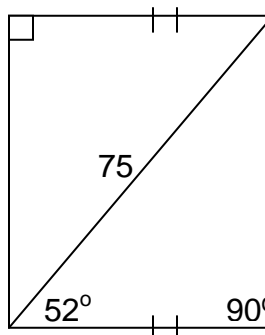
b.



c.



d.



Congruent sides are also parallel.  
Perimeter of quadrilateral = \_\_\_\_\_

## Applications of Trigonometry (pp. 1 of 3) **KEY**

Trigonometric ratios have many real world applications from surveying to navigation where calculation of distances and angles might be required.

Trigonometric ratios can be applied in any situation involving a right triangle where only one acute angle and one side are known.

- Draw a diagram of the right triangle.
- Determine which trig ratio will work best with the given data.
- Solve the problem.

### Examples

1. A 25-foot ladder is propped against a storeroom. The angle the ladder forms with the ground is  $50^\circ$ . How far up the storeroom does the ladder reach?  
**19.15 ft.**
2. A pine tree creates a shadow of 55 feet when the sun is at an angle of elevation of  $42^\circ$ . How tall is the pine tree?  
**49.52 ft.**
3. The angle of depression from the top of a 121-foot lighthouse to a ship coming in to harbor is  $16^\circ$ . To the nearest foot, how far out is the ship from the lighthouse?  
**422 ft.**

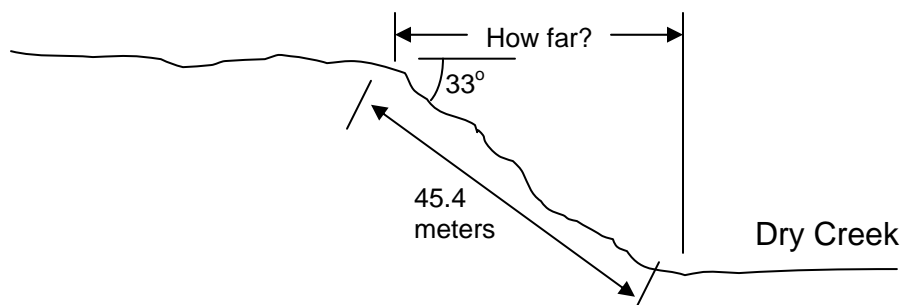
## Applications of Trigonometry (pp. 2 of 3) **KEY**

### Practice Problems

1. Commercial airliners fly at an altitude of about 10 kilometers. They start descending toward the airport when they are still far away in order to lessen the angle they must drop.
  - a. If the pilot wants the path to make a  $5^\circ$  angle with the ground, how far out must the plane begin to descend? **114 km**
  - b. If he starts descending 350 kilometers from the airport, what angle will the plane's path make with the ground?  
 **$1.6^\circ$**
2. A new rope must be ordered for the flagpole in front of the school. Before ordering the rope, the height of the pole must be determined. It is observed that the flagpole casts a shadow 10.5 meters long when the sun is at an angle of elevation of  $33^\circ$ . How tall is the flagpole?  
**6.82 m**
3. A cat is trapped on a tree branch 18.5 feet above the ground. The ladder is only 20 feet long. If you place the ladder's tip on the branch, what angle must the ladder make with the ground?  
 **$67.7^\circ$**
4. San Francisco has very steep streets. Sue decides to determine the angle the street she lives on makes with the horizontal. On the wall of her house she measures horizontal and vertical distances of 33 centimeters and 5 centimeters, respectively. What angle does her street make with the horizontal?  
 **$8.6^\circ$**

## Applications of Trigonometry (pp. 3 of 3) **KEY**

5. When surveyors measure distances on land that slopes significantly, the distance measured along the ground is longer than the horizontal distance to be drawn on maps. Trigonometry is used to calculate this horizontal distance. Suppose the top edge of Dry Creek is 45.4 meters from the bottom and the land slopes downward at an angle of  $33^\circ$ . How far is the horizontal distance to the bottom of Dry Creek?



**38.1 m**

6. From a point on the North Rim of the Grand Canyon to a point on the South Rim, a surveyor measures an angle of depression of  $3^\circ$ . The horizontal distance between the two points is 10 miles. How many **feet** is the South Rim below the North Rim?

**0.5240777....miles or  $(0.52407779...)(5280 \text{ ft/mile}) = 2767.13...ft.$**

## Applications of Trigonometry (pp. 1 of 3)

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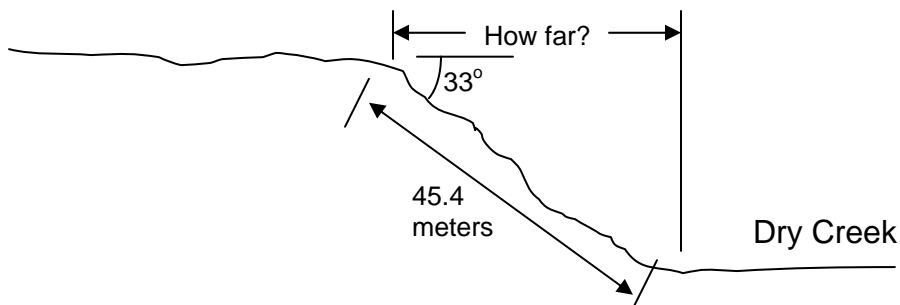
## Applications of Trigonometry (pp. 2 of 3)

### Practice Problems

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## Applications of Trigonometry (pp. 3 of 3)

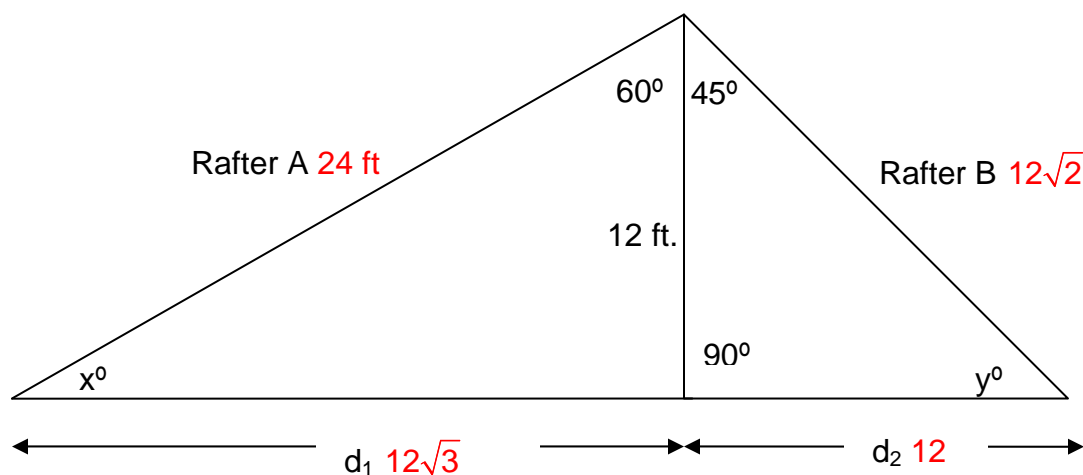
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## Roof Pitch Revisited **KEY**

You are supervising a construction project when one of your carpenters shows you a figure from the blueprints for the project. He states that the figure does not contain enough information to determine the materials needed to construct a section of roof. You decide to take a look and see the figure below.



- Use your knowledge of special right triangles to solve for the following missing pieces. Explain your reasoning. **See figure above.**
  - The missing angle values. **Justified by Triangle Angle Sum Theorem.**
  - The distances  $d_1$  and  $d_2$ . **Justified by  $30^\circ$ - $60^\circ$ - $90^\circ$  and  $45^\circ$ - $45^\circ$ - $90^\circ$  Theorems.**
  - The lengths of Rafter A and Rafter B. **Justified by  $30^\circ$ - $60^\circ$ - $90^\circ$  and  $45^\circ$ - $45^\circ$ - $90^\circ$  Theorems.**

- Use your knowledge of trigonometric ratios to verify and justify your solutions from problem 1 above. Explain your reasoning.

**Angles justified by Triangle Angle Sum Theorem.**

$$d_1 = 12 \tan 60^\circ$$

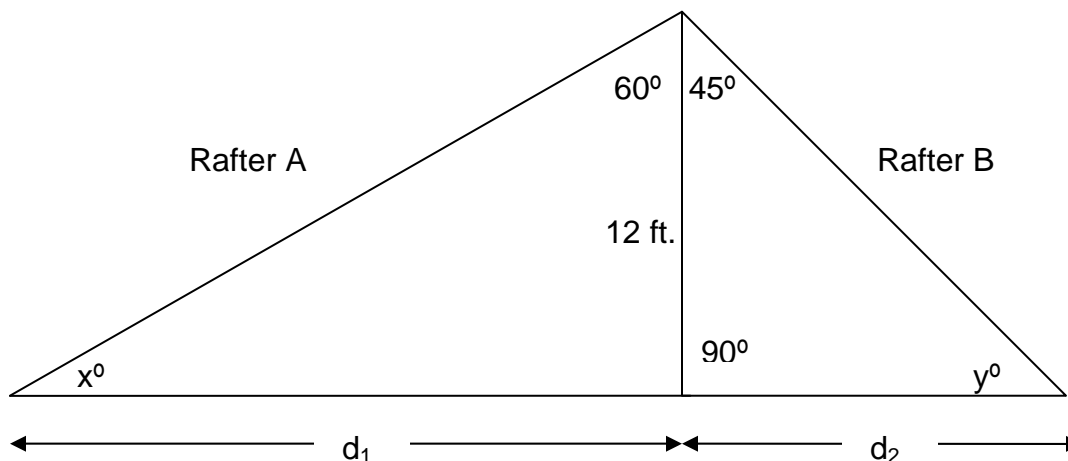
$$d_2 = 12 \tan 45^\circ$$

$$\text{Rafter A} = \frac{12}{\cos 60^\circ}$$

$$\text{Rafter B} = \frac{12}{\cos 45^\circ}$$

## Roof Pitch Revisited

You are supervising a construction project when one of your carpenters shows you a figure from the blueprints for the project. He states that the figure does not contain enough information to determine the materials needed to construct a section of roof. You decide to take a look and see the figure below.



1. Use your knowledge of special right triangles to solve for the following missing pieces. Explain your reasoning.
  - a. The missing angle values.
  - b. The distances  $d_1$  and  $d_2$ .
  - c. The lengths of Rafter A and Rafter B.
  
2. Use your knowledge of trigonometric ratios to verify and justify your solutions from problem 1 above. Explain your reasoning.